CHAPTER 1

BASICS OF CARTAN GEOMETRY

1. TO $B^{(3)}$ OR NOT TO $B^{(3)}$: THE ADVENT OF DISCOVERY

Discovery of the $B^{(3)}$ field in 1991 has its roots in

X[1]
X
X

If Maxwell's equations are formulated relativistically with $\sigma \neq 0$ in terms of a covariant Dirac polarized vacuum with charge, j = (j, j_0) where $j_0 \neq 0$, $j_0 \sim \bar{\rho}$ then both $\nabla \cdot E \neq 0$ and $\nabla \times E \neq 0$. The nonzero j_0 relates to the magnetic field density $B^{(0)}$ in the original Evans-Vigier $B^{(3)}$ field theory [x,x,x] where the fields $B^{(1),}$ $B^{(2)}$ and $B^{(3)}$ satisfy the cyclic relations

$$\begin{aligned} B^{(1)} \times B^{(2)} &= iB^{(0)}B^{(3)*} = iB^{(0)}B^{(3)} \\ B^{(2)} \times B^{(3)} &= iB^{(0)}B^{(1)*} = iB^{(0)}B^{(2)} \\ B^{(3)} \times B^{(1)} &= iB^{(0)}B^{(2)*} = iB^{(0)}B^{(1)} \end{aligned} \tag{x}$$

Evans determined that a putative $B^{(3)}$ field suggests photon mass, m_γ as shown utilizing the Proca equation [x].

1.1 HISTORICAL BACKGROUND

Geometry was equated with beauty by the ancient Greeks, and was used by them to create art of the highest order. The Parthenon for example was built on principles of geometry, and a deliberate flaw introduced so as not to offend the gods with perfection. A thousand years later the Book of Kells scaled the magnificent peak of insular Celtic art, using the principles of geometry to draw the fine triskeles. Aristotelian thought dominated natural philosophy until Copernicus placed the sun at the centre of the solar system, a challenge to Ecclesia, the dominant European power that had grown out of the beehive cells of remote places such as Skellig Michael. In such places civilization had clung on by its fingernails after the Roman empire was swept away by vigorous peoples of the far north. They had their own type of geometry carved on the prows of their ships, interwoven patterns carved in wood. Copernicus offered a challenge to dogma, always a dangerous thing to do, and human nature never changes. Gradually a new enlightenment began to dawn, with figures such as Galileo and

[1] xxx

Kepler at its centre. Leonardo da Vinci in the early renaissance had sensed that nature is geometry, and that one cannot do physics without mathematics. Earlier still, the perpendicular and gothic styles of architecture resulted in great European cathedrals built on geometry, for example Cluny, Canterbury and Chartres. Both Leonardo and Descartes thought in terms of swirling whirlpools, reminiscent of van Gogh's starry night.

Francis Bacon thought that nature is the measuring stick of all theory, and that dogma is ultimately discarded. This was another challenge to Ecclesia. Galileo boldly asserted that the sun is at the centre of the solar system as we call it today. That offended Ecclesia so he was put under house arrest but survived. It is dangerous to challenge dogma, to challenge the comfortable received wisdom which by passes the need to think. So around 1600, as Bruno was burnt at the stake, Kepler began the laborious task of analyzing the orbit of Mars. Tycho Brahe has finally given him the needed data. This is all described in Koestler's famous book, "The Sleepwalkers". Kepler used the ancient thought in a new way, geometry describes nature, nature is geometry. The orbit of Mars was found to be an ellipse, not a circle, with the sun at one of its foci. After an immense amount of work, Kepler discovered three laws of planetary motion. These laws were synthesized by Newton in his theory of universal gravitation, later developed by many mathematicians such as Euler, Bernoulli, Laplace and Hamilton.

All of these descriptions of nature rested on three dimensional space and time. The three dimensional space was that of Euclid and time flowed forward on its own. Space and time were different entities until Michelson and Morley carried out an experiment which overturned this dogma. It seemed that the speed of light, c was independent of the direction in which it was measured. It seemed that c was an upper limit, a velocity, v could not be added to c. Fitzgerald and Heaviside corresponded about this puzzling result and Heaviside came close to resolving the contradiction. Lorentz swept away the dogma of two thousand years by merging three dimensional (3D) space with time to create spacetime in four dimensions, (ct, X, Y, Z). This was the beginning of the theory of special relativity, in transforming quantities from one frame to another, c remained constant but t, X, Y, and Z varied, so quantities in the new frame are (ct', X', Y', Z'). Lorentz considered the simple case when one frame moves with respect to the other at a constant velocity v but if one frame accelerated with respect to the other the theory became untenable. This is the famous Lorentz transform. The spacetime used by Lorentz is known as flat spacetime, meaning that it is described by a certain limit of a more general geometry. Flat spacetime is described by a simple metric known as diag. (1, -1, -1, -1), a 4 x 4 matrix with these numbers on its diagonal. Lorentz, Poincaré, Voigt and many others applied the theory of special relativity to electrodynamics and found that the Maxwell Heaviside equations obey the Lorentz transform, and were therefore thought to be equations of special relativity. The Newtonian system of dynamics does not obey the Lorentz transform, there is no limit on the linear velocity in the Newtonian system.

So there developed a schism between dynamics and electrodynamics, they seemed to obey different transformation laws and different geometries. Dynamics had been described for two centuries since Newton by the best minds as existing in Euclidean space and time. Electrodynamics existed in flat spacetime. The underlying geometries of the two subjects seemed to be different. Attempts were made around the turn of the twentieth century to resolve this fundamental challenge to physics. Einstein in 1905 applied the principles of Lorentz to dynamics, using the concepts of four momentum, relativistic momentum and energy. The laws of dynamics were merged with the laws of electrodynamics using c as a universal constant. Einstein also challenged dogma and many scientists of the old school rejected special relativity out of hand. Some dogmatists still reject it. From 1905 onwards physics ceased to be comprehensible without mathematics, which is why so few people understand physics today and are easily deceived by dogmatists. At the end of the nineteenth century several other flaws were found in the older physics, and these were resolved by quantum mechanics, notably by Planck's quantization of energy. Quantum mechanics seemed to give an accurate description of black body radiation, the photoelectric effect and the specific heat of solids, but departed radically from classical physics. Many people today do not understand quantum mechanics or special relativity because they are completely counter intuitive. Planck, Einstein and many others, notably Sommerfeld and his school, developed what is known as the old quantum theory.

The old quantum theory and special relativity had many successes, but existed as separate theories. There was no geometrical framework with which the two types of theory could be unified and special relativity was restricted to one frame moving with respect to another with a constant velocity. The brilliant successes of the classical Newtonian physics were thought of as a limit of special relativity, one in which the velocity, v of a particle is $\ll c$. A new corpuscular theory of light emerged in the old quantum theory, and this corpuscle was named the photon about twenty years later. Initially the photon was thought of as quantized electromagnetic radiation. In about 1905 physics was split three ways, and the work of Rutherford and his school began to show the existence of elementary particles, the electron having been just discovered. Einstein, Langevin and others analyzed the Brownian motion to show the existence of molecules, first inferred by Dalton. The old dogmatists had refused to accept the existence of molecules for over a century. The Rutherford group showed the existence of the alpha particle and inferred the existence of the nucleus and the neutron, later discovered by Chadwick. Rutherford and Soddy demonstrated the existence of isotopes, nuclei with the same number of protons but different number of neutrons. So physics rapidly diverged in all directions, there was no unified theory that could explain all of these tremendous discoveries.

Geometry in the meantime had developed away from Euclidean principles. There were many contributors, the most notable achievement of the mid nineteenth century was that of Riemann, who proposed the concept of metric. Christoffel inferred the geometrical connection shortly thereafter. The metric and the connection describe the difference between Euclidean geometry and a new type of geometry often known as Riemannian geometry. In fact Riemann inferred only the metric. The curvature tensor or Riemann tensor was inferred much later in about 1900 by Ricci and his student Levi Civita. It took over thirty years to progress from the metric to the curvature tensor. There was no way of knowing the symmetry of the connection. The latter has one upper index and two lower indices, so is a matrix for each upper index. In general a matrix is asymmetric, can have any symmetry, but can always be written as the sum of a symmetric matrix and an antisymmetric matrix. So the connection for each upper index is in general the sum of symmetric and antisymmetric components. Christoffel, Ricci and Levi Civita assumed without proof that the connection is symmetric in its lower two indices - the symmetric connection. This assumption was used by Bianchi in about 1902 to prove the first Bianchi identity from which the second Bianchi identity follows. Both these identities assume a symmetric connection. The antisymmetric part of the connection was ignored irrationally, or dogmatically. This dogma eventually evolved into general relativity, an incorrect dogma which unfortunately influenced thought in natural philosophy for over a century.

The first physicist to take much notice of these developments in geometry appears to have been Einstein, whose friend Grossmann was a mathematician. Einstein was not fond of the complexity of the Riemannian geometry as it became known, and never developed a mastery of the subject. After several attempts from 1905 to 1915 Einstein used the second Bianchi identity and the covariant Noether Theorem to deduce a field equation of general relativity in late 1915. This field equation was solved by Schwarzschild in December 1915, but Schwarzschild heavily criticised its derivation. It was later criticised by Schrödinger, Bauer, Levi-Civita and others, notably Elie Cartan.

Cartan was among the foremost mathematicians of his era and inferred spinors in 1913. In the early twenties he used the antisymmetric connection to infer the existence of torsion, a quantity that had been thrown away twenty years earlier by Ricci, Levi-Civita and Bianchi, and also by Einstein. The entire theory of general relativity continued to neglect torsion throughout the twentieth century. Cartan and Einstein corresponded but never really understood each other. Cartan realized that there are two fundamental quantities in geometry, torsion and curvature. He expressed this with Maurer in the form of two structure equations and using a differential geometry developed to try to merge the concept of spinors with that of torsion and curvature. The structure equations were still almost unknown to physics before they were implemented in 2003 in the subject of this book , the Einstein Cartan Evans unified field theory, known as ECE theory. The ECE theory has swept the world of physics, and has been read an accurately estimated thirty to fifty million times in a decade. This phenomenon is known as the post Einstein paradigm shift, a phrase coined by Alwyn van der Merwe.

The first and second Maurer Cartan structure equations can be translated into the Riemannian definitions of respectively torsion and curvature. The concept of commutator of covariant derivatives has been developed to give the torsion and curvature simultaneously with great elegance. The commutator acts on any tensor in any space of any dimension and always isolates the torsion simultaneously with the curvature. The torsion is made up of the difference of two antisymmetric connections, and these connections have the same antisymmetry as the commutator. The connection used in the curvature is also antisymmetric. A symmetric connection means a symmetric commutator. A symmetric commutator always vanishes, and the torsion and curvature vanish if the connection is symmetric. This means that the second Bianchi identity used by Einstein is incorrect and that his field equation is meaningless.

The opening sections of this book develop this basic geometry and use the Cartan identity to produce the geometrically correct field equations of electrodynamics unified with gravitation. The dogmatists have failed to achieve this unification because they used a symmetric connection and because they continued to regard electrodynamics as special relativity.

1.2 THE STRUCTURE EQUATIONS OF MAURER AND CARTAN

These structure equations were developed using the notation of differential geometry and are defined in many papers [1-10] of the UFT series on www.aias.us. The most important discovery made by Elie Cartan in this area of his work was that of spacetime torsion. In order for torsion to exist the geometrical connection must be antisymmetric. In the earlier work of Christoffel, Ricci, Levi-Civita and Bianchi the connection had been assumed to be symmetric. The Einsteinian general relativity continued to repeat this error for over a hundred years, and this incorrect symmetry is the reason why Einstein did not succeed in developing a unified field theory, even though Cartan had informed him of the existence of torsion. The first structure equation defines the torsion in terms of differential geometry. In the simplest or minimalist notation the torsion, T is:

$$T = D \wedge q = d \wedge q + \omega \wedge q \tag{1}$$

Where $d \wedge$ denotes the wedge derivative of differential geometry, q denotes the Cartan tetrad and ω denotes the spin connection of Cartan. The symbol $D \wedge$ defines the covariant wedge derivative. In this notation the indices of differential geometry are omitted for clarity. The Cartan tetrad was also known initially as the *vielbein* (many legged) or *vierbein* (4-legged).

The wedge derivative is an elegant formulation that can be translated [1-11] into tensor notation. This is carried out in full detail in the UFT papers, which can be consulted using indices or with Google. In this section we concentrate on the essentials without overburdening the text with details. The spin connection is related to the Christoffel connection.

The only textbook to even mention torsion in a clear, understandable way is that of S. M. Caroll [11], accompanied by online notes. The ECE theory uses the same geometry precisely as that described in the first three chapters of Carroll, but ECE has evolved completely away from the interpretation given by Carroll in his chapter four onwards. Carroll defines torsion but then neglects it without reason, and this is exactly what the twentieth century general relativity proceeded to do. All of Carroll's proofs have been given in all detail in the UFT papers and books [1-10] and a considerable amount of new geometry also inferred, notably the Evans identity. In Carroll's notation the first structure equation is:

$$T^a = d \wedge q^a + \omega^a b \wedge q^b \tag{2}$$

in which the Latin indices of the tetrad and spin connection have been added. These indices were originally indices of the tangent Minkowski spacetime defined by Cartan at a point, P of the general base manifold. The latter is defined with Greek indices. Eq. (2) when written out more fully becomes:

$$T^a_{\mu\nu} = \left(d \wedge q^a\right)_{\mu\nu} + \omega^a_{\mu b} \wedge \nu q^b_\nu . \tag{3}$$

So the torsion had one upper Latin index and two lower Greek indices. It is a vector valued two form of differential geometry which is by definition antisymmetric in its Greek indices:

$$T^a_{\mu\nu} = -T^a_{\nu\mu} \tag{4}$$

The torsion is a rank three mixed index tensor.

The tetrad has one upper Latin index, a and one lower Greek index, μ . It is a vector valued 1-form of differential geometry and is a mixed index rank 2-tensor. The tetrad is defined as a matrix relating a vector, V^a and a vector, V^μ :

$$V^a = q^a_\mu V^\mu \tag{5}$$

In his original work Cartan defined V^a as a vector in the tangent spacetime of a base manifold, and defined the vector, V^μ in the base manifold. However, during the course of development of ECE theory it was inferred that the tetrad can be used more generally as shown in great detail in the UFT papers to relate a vector, V^a defined by a given curvilinear coordinate system to the same vector defined in another curvilinear coordinate system, for example cylindrical polar and Cartesian, or complex circular and Cartesian. The spin connection has one upper and one lower Greek index and one lower Latin index and is related to the Christoffel connection through a fundamental theorem of differential geometry known obscurely as the tetrad postulate. The tetrad postulate is the theorem which states that the complete vector field in any space in any dimension is independent of the way in which that complete vector field is written in terms of components and basis elements. For example in 3D the complete vector field is the same in cylindrical polar and Cartesian coordinates or any curvilinear coordinates. The Christoffel connection does not transform as a tensor [1-11], so the spin connection is not a tensor, but for some purposes may be defined as a 1-form, with one lower Greek index.

The wedge product of differential geometry is precisely defined in general, and translates Eq. (3) in to tensor notation by acting on the 1-form, q^a_μ and the 1-form, $\omega^a_{\mu b}$ to give:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial \nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \tag{6}$$

which is a tensor equation. It is seen that the entire equation is antisymmetric in the Greek indices, μ and ν, which means that:

$$T^a_{\mu\nu} = \partial_\nu q^a_\mu - \partial_\mu q^a_\nu + \omega^a_{\nu b} q^b_\mu - \omega^a_{\mu b} q^b_\nu . \tag{7}$$

This result is important for the ECE antisymmetry laws developed later in this book. In this tensor equation there is summation over repeated indices, so:

$$\omega^a_{\mu b} q^b_\nu = \omega^a_{\mu 1} q^1_\nu + \ldots + \omega^a_{\mu\nu} q^n_n \tag{8}$$

in general. It is seen that the torsion has some resemblance to the way in which an electromagnetic field was defined by Lorentz, Poincaré and others in terms of the 4-potential, a development of the work of Heaviside. This led to the inference of ECE theory in 2003 through a simple postulate described in the next chapter. The difference is that the torsion contains an upper index, a and contains an antisymmetric term in the spin connection.

All the equations of Cartan geometry are generally covariant, which means that they transform under the general coordinate transformation, and are equations of general relativity. Therefore the torsion is generally covariant as required by general relativity. The tetrad postulate results in the following relation between the spin connection and the gamma connection:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu = \Gamma^\lambda_{\mu\nu} q^a_\lambda \tag{9}$$

and using this equation in Eq. (6) gives the Riemannian torsion:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{10}$$

In deriving the Riemannian torsion the following equation of Cartan geometry has been used:

$$T^a_{\mu\nu} = q^a_\lambda T^\lambda_{\mu\nu} \tag{11}$$

which means that the tetrad plays the role of switching the a index to a λ index. Similarly the equation for torsion can be simplified using:

$$\omega^a_{\mu b} q^b_\nu = \omega^a_{\mu\nu} ; \;\; \omega^a_{\nu b} q^b_\mu = \omega^a_{\nu\mu} \tag{12}$$

to give a simpler expression

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu\nu} - \omega^a_{\nu\mu} . \tag{13}$$

It can be seen that the Riemannian torsion is antisymmetric in μ and ν so $T^{\lambda}_{\mu\nu}$ vanishes if the connection were symmetric, if the following were true:

$$\Gamma^{\lambda}_{\mu\nu} = ?\Gamma^{\lambda}_{\nu\mu} \tag{14}$$

The Einsteinian general relativity always assumed Eq. (14) without proof. In fact the commutator method to be described below proves that the connection is antisymmetric. We arrive at the conclusion that Einsteinian general relativity is refuted entirely by its neglect of torsion, and part of the purpose of this book is to forge a new cosmology based on torsion. In order to make the theory of torsion of use to engineers and chemists the tensor notation needs to be translated to vector notation. The precise details of how this is done are given again in the UFT papers and other material on www.aias.us.

In vector notation the torsion spits into orbital torsion and spin torsion. In order to define these precisely the tetrad 4-vector is defined as the 4-vector:

$$q^a_\mu = \left(q^a_0, -\underline{q}^a\right), \tag{15}$$

$$q^{a\mu} = \left(q^{a0}, \underline{q}^a\right), \tag{16}$$

with a timelike component q^a_0 and a spacelike component $\underline{q}^a$. Similarly the spin connection is defined as the four vector:

$$\omega^a_{\mu b} = \left(\omega^a_{0b}, -\underline{\omega}^a_b\right). \tag{17}$$

In this notation the orbital torsion is:

$$\underline{T}^a{}_{orb} = -\underline{\nabla} q^a_0 - \frac{1}{c}\frac{\partial \underline{q}^a}{dt} - \omega^a_{0b}\underline{q}^b + \underline{\omega}^a_b q^b_0 \tag{18}$$

and the spin torsion is:

$$T^a_{spin} = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a_b \times \underline{q}^b$$

(19)

In ECE electrodynamics the orbital torsion gives the electric field strength and the spin torsion gives the magnetic flux density. In ECE gravitation part of the orbital torsion gives the acceleration due to gravity, and the spin torsion gives the magnetogravitational field. The physical quantities of electrodynamics and gravitation are obtained directly from the torsion and directly from Cartan geometry. For example the fundamental $B^{(3)}$ field of electrodynamics [1-11] is obtained from the spin torsion of the first structure equation.

In minimal notation the second Cartan Maurer structure equation defines the Cartan curvature:

$$R = D \wedge \omega = d \wedge \omega + \omega \wedge \omega \tag{20}$$

so the torsion is the covariant wedge derivative of the tetrad and the curvature is the covariant wedge derivative of the spin connection. Fundamentally therefore these are simple definitions, and that is the elegance of Cartan's geometry. When expanded out into tensor and vector notation they look much more complicated but convey the same information. In the standard notation of differential geometry Eq. (20) becomes:

$$R^a_b = d \wedge \omega^a_b + \omega^a_c \wedge \omega^c_b \tag{21}$$

where there is summation over repeated indices. When written out in full Eq. (21) becomes:

$$R^a_{b\mu\nu} = \left(d \wedge \omega^a_b\right)_{\mu\nu} + \omega^a_{\mu c} \wedge \omega^c_{\nu b} \tag{22}$$

where the indices of the base manifold have been reinstated. In tensor notation Eq. (22) becomes:

$$R^a_{b\mu\nu} = \partial_\mu \omega^a_{\nu b} - \partial_\nu \omega^a_{\mu b} + \omega^a_{\mu c}\omega^c_{\nu b} - \omega^a_{\nu c}\omega^c_{\mu b} \tag{23}$$

which defines the Cartan curvature as a tensor valued 2-form. It is tensor valued because it has indices *a* and *b*, and is a differential two form [1-11] antisymmetric in μ and ν. Using the tetrad postulate (9) it can be shown that Eq. (24) is equivalent to the Riemann curvature tensor:

$$R^\lambda_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho} \tag{24}$$

first inferred by Ricci and Levi Civita in about 1900. The proof of this is complicated but is given in full in the UFT papers.

The geometrical connection was inferred by Christoffel in the 1860s in order to define a generally covariant derivative. In 4D for example the ordinary derivative, ∂_μ does not transform covariantly [1-11] but by definition the covariant derivative of any tensor has this property. The Christoffel connection is defined by:

$$D_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda}\partial_\mu V^\lambda \tag{25}$$

and the spin connection is defined by:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b . \tag{26}$$

Without additional information there is no way in which to determine the symmetry of the Christoffel and spin connection, and both are asymmetric in general in their lower two indices. The covariant derivative can act on any tensor of any rank in a well-defined manner explained in full detail in the UFT papers on www.aias.us. When it acts on the tetrad, a rank 2 mixed index tensor, it produces the result [1-11]:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda . \tag{27}$$

The tetrad postulate means that:

$$D_\mu q^a_\nu = 0 \tag{28}$$

and so the covariant derivative of the tetrad vanishes in order to maintain the invariance of the complete vector field. This has been a fundamental theorem of Cartan geometry for almost a hundred years. The tetrad postulate is the theorem by which Cartan geometry is translated into Riemann geometry.

The Riemann torsion and Riemann curvature are defined elegantly by the commutator of covariant derivatives. This is an operator that acts on any tensor in any space of any dimension. When it acts on a vector it is defined for example by:

$$\left[D_\mu,\ D_\nu\right]V^\rho = D_\mu\left(D_\nu V^\rho\right) - D_\nu\left(D_\mu V^\rho\right) \tag{29}$$

As shown in all detail in UFT 99 Eq. (29) results in:

$$\left[D_\mu,\ D_\nu\right]V^\rho = R^\rho_{\mu\nu\sigma}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho. \tag{30}$$

The Riemann curvature and Riemann torsion are always produced simultaneously by the commutator, which therefore produces the first and second Cartan Maurer structure equations when the tetrad postulate is used to translate the Riemann torsion and Riemann curvature to the Cartan torsion and Cartan curvature. The commutator also defines the antisymmetry of the connection and this is of key importance. By definition the commutator is antisymmetric in the indices μ and ν:

$$\left[D_\mu,\ D_\nu\right]V^\rho = -\left(D_\nu,\ D_\mu\right)V^\rho \tag{31}$$

and vanishes if these indices are the same, i.e. if the connection is symmetric. From inspection of the equation:

$$\left[D_\mu,\ D_\nu\right]V^\rho = -\left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right)D_\lambda V^\rho + R^\rho_{\mu\nu\sigma}V^\sigma \tag{32}$$

the connection has the same symmetry as the commutator, so the connection is antisymmetric:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \tag{33}$$

a result of key importance. A symmetric connection means a null commutator and this means that the Riemann torsion and Riemann curvature both vanish if the connection is symmetric.

The Einsteinian general relativity used a symmetric connection incorrectly, so the entire twentieth century era is refuted. This is the essence of the post Einsteinian paradigm shift. The correct general relativity is based on field equations obtained from Cartan geometry. These field equations are obtained from identities of Cartan Geometry. The first such identity in minimal notation is:

$$D \wedge T = d \wedge T + \omega \wedge T :\, = R \wedge q = q \wedge R \tag{34}$$

and this is referred to in this book as the Cartan identity. The covariant derivative of the torsion is the wedge product of the tetrad and curvature. The wedge products in Eq. (34) are those of a 1-form and a 2-form. In the UFT papers it is shown that this produces the following result in tensor notation:

$$\begin{aligned}&\partial_\mu T^a_{\nu\rho} + \partial_\rho T^a_{\mu\nu} + \partial_\nu T^a_{\rho\mu} + \omega^a_{\mu b}T^b_{\nu\rho} + \omega^a_{\rho b}T^b_{\mu\nu} + \omega^a_{\nu b}T^b_{\rho\mu} :\\ &= R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu}\end{aligned} \tag{35}$$

a sum of three terms. In papers such as UFT 137 this identity is proven in complete detail using the tetrad postulate. The proof is complicated but again shows the great elegance of the Cartan geometry. Using the concept of the Hodge dual [1-11] the result (35) can be expressed as:

$$\partial_\mu \tilde{T}^{a\mu\nu} + \omega^a_{\mu b} \tilde{T}^{b\mu\nu} := \tilde{R}^{a\mu\nu}_{\mu} \tag{36}$$

where the tilde's denote the tensor that is Hodge dual to $T^a_{\mu\nu}$. In 4D the Hodge dual of an antisymmetric tensor, or 2-form, is another antisymmetric tensor. From Eq. (36) the Cartan identity can be expressed as:

$$\partial_\mu \tilde{T}^{a\mu\nu} = j^{a\nu} = \tilde{R}^{a\mu\nu}_{\mu} - \omega^a_{\mu b} \tilde{T}^{b\mu\nu} \tag{37}$$

Defining:

$$j^{a\nu} = \left(j^{a0},\ \underline{j}^a \right) \tag{38}$$

the Cartan identity splits into two vector equations:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a{}_{spin} = j^{a0} \tag{39}$$

and

$$\frac{1}{c} \frac{\partial \underset{\sim}{T}^a{}_{spin}}{\partial t} + \underline{\nabla} \times \underset{\sim}{T}^a{}_{orb} = \underline{j}^a \tag{40}$$

These become the basis for the homogeneous equations of electrodynamics in ECE theory, and define the magnetic charge current density in terms of geometry. These equations are given in the Engineering Model of ECE theory on www.aias.us. They also define the homogeneous field equations of gravitation.

The Evans identity of differential geometry was inferred during the course of the development of ECE theory and in minimal notation it is:

$$D \wedge \tilde{T} = d \wedge \tilde{T} + \omega \wedge \tilde{T} := \tilde{R} \wedge q = q \wedge \tilde{R} \tag{41}$$

It is valid in 4D, because the Hodge dual of a 2-form in 4D is another 2-form. So the Hodge duals of the torsion and curvature obey the Cartan identity. This result is the Evans identity (41). In tensor notation it is:

$$\begin{aligned} &\partial_\mu \tilde{T}^a_{\nu\rho} + \partial_\rho \tilde{T}^a_{\mu\nu} + \partial_\nu \tilde{T}^a_{\rho\mu} + \omega^a_{\mu b} \tilde{T}^b_{\nu\rho} + \omega^a_{\rho b} \tilde{T}^b_{\mu\nu} + \omega^a_{\nu b} \tilde{T}^b_{\rho\mu} : \\ &= \tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} \end{aligned} \tag{42}$$

an equation which is equivalent to:

$$\partial_\mu T^{a\mu\nu} + \omega^a_{\mu b} T^{b\mu\nu} := R^{a\mu\nu}_{\mu} \tag{43}$$

as shown in full detail in the UFT papers. The tensor equation (42) splits in to two vector equations:

$$\underline{\nabla} \cdot \underline{T}^a{}_{orb} = J^{a0} = R^{a\mu 0}_{\mu} - \omega^a_{\mu b} T^{b\mu 0} \tag{44}$$

and

$$\underline{\nabla} \times \underset{\sim}{T}^a_{spin} - \frac{1}{c}\frac{\partial \underset{\sim}{T}^a_{orb}}{\partial t} = \underline{J}^a \tag{45}$$

When translated into electrodynamics these become the inhomogeneous field equations, which define the electric charge density and the electric current density in terms of geometry.

If torsion is neglected or incorrectly assumed to be zero, the Cartan identity reduces to

$$R \wedge q = 0, \tag{46}$$

which is the elegant Cartan notation for the first Bianchi identity:

$$R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\nu\rho\mu} = 0. \tag{47}$$

The second Bianchi identity can be derived from the first Bianchi identity and is

$$D_\mu R^k_{\lambda\nu\rho} + D_\rho R^k_{\lambda\mu\nu} + D_\nu R^k_{\lambda\rho\mu} = 0. \tag{48}$$

Clearly the two Bianchi identities are true if and only if the torsion is zero. In other words the two identities are true if and only if the Christoffel connection is symmetric. The commutator method shows that the Christoffel connection is antisymmetric so the two Bianchi identities are incorrect. The first Bianchi identity must be replaced by the Cartan identity (34) and the second Bianchi identity was replaced in UFT 255 by:

$$\begin{aligned} &D_\mu D_\lambda T^k_{\nu\rho} + D_\rho D_\lambda T^k_{\mu\nu} + D_\nu D_\lambda T^k_{\rho\mu} :=\\ &D_\mu R^k_{\lambda\nu\rho} + D_\rho R^k_{\lambda\mu\nu} + D_\nu R^k_{\lambda\rho\mu}. \end{aligned} \tag{49}$$

Therefore Einstein used entirely the wrong identity (47) in his field equation. No experiment can prove incorrect geometry, and indeed the claims of experimentalists to have tested the Einstein field equation with precision have been extensively criticised for many years. The contemporary experimental data themselves may or not be precise, but they do not prove incorrect geometry. Einstein effectively threw away the first Cartan Maurer structure equation, so his geometry contained and still contains only half of the geometrical truth, and geometry is the most self-contained of all subjects. The velocity curve of the whirlpool galaxy, discovered in the late fifties, entirely and completely refutes both Einstein and Newton. In several of the UFT papers on www.aias.us, the velocity curve is explained straightforwardly by ECE theory using again the minimum of postulates, for example UFT 238. The dogmatists used and still use ad hoc ideas such as dark matter to cover up the catastrophic failure of the Einstein and Newton theories in whirlpool galaxies. They became idols of the cave, and dreamt up dark matter in its darkest corners. Their claim that the universe is made up mostly of dark matter is an admission of abject failure. To compound this failure they still claim that the Einstein theory is very precise in places such as the solar system. This dogma has reduced natural philosophy to utter nonsense. Either a theory works or it does not work. It cannot be brilliantly successful and fail completely at the same time. ECE and the post Einsteinian paradigm shift uses no dark matter and no ideas deliberately cobbled up so they cannot be tested experimentally: "*not even wrong*" as Pauli wrote.

In some recent work in UFT 254 onwards the Cartan identity has been reduced to a simple and clear vectorial format

$$\underline{\nabla} \cdot \underline{\omega}^a_b \times \underset{\sim}{T}^b{}_{spin} := \underline{\omega}^a_b \cdot \underline{\nabla} \times \underset{\sim}{T}^b{}_{spin} - \underset{\sim}{T}^b{}_{spin} \cdot \underline{\nabla} \times \underline{\omega}^a_b . \tag{50}$$

As always in ECE theory this vector identity is generally covariant. It is very useful when used with the geometrical equations for magnetic and electric charge current densities also developed in UFT 254 onwards. In the following chapter it is shown that combinations of ECE equations such as these produce many new insights.

This introductory survey of Cartan geometry has shown that the ECE theory is based entirely on four equations: the first and second Cartan Maurer structure equations, the Cartan identity, and the tetrad postulate. These equations have been known and taught for almost a century. Using these equations the subject of natural philosophy has been unified on a well-known geometrical basis. Electromagnetism has been unified with gravitation [x] and new methods developed to describe the structure of elementary particles. General relativity has been unified with quantum mechanics by developing the tetrad postulate into a generally covariant wave equation:

$$\left(\square + K^2\right) q^a_\mu = 0 \tag{51}$$

where

$$K^2 = q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right). \tag{52}$$

The wave equation (51) has been reduced to all the main relativistic wave equations such as the Klein Gordon, Proca and Dirac wave equations, and in so doing these wave equations have been derived as equations of general relativity. They are all based on the most fundamental theorem of Cartan geometry, the tetrad postulate. The Dirac equation has been developed into the fermion equation by factorizing the ECE wave equation that reduces in special relativity to the Dirac wave equation. The fermion equation needs only two by two matrices, and does not suffer from negative energy while at the same time producing the positron and other antiparticles. So the discoveries of the Rutherford group have also been explained geometrically.

The Heisenberg Uncertainty Principle was replaced and developed in UFT 13, and easily shown to be incorrect in UFT 175. The uncertainty principle should be described more accurately as the indeterminacy principle, which is an admission of failure from the outset. It was rejected by Einstein, de Broglie, Schrödinger and others at the famous 1927 Solvay Conference and split natural philosophy permanently into scientists and dogmatists. The indeterminacy principle has been experimentally proven to be wildly wrong by the Croca group [12] using advanced microscopy and other experimental methods. The dogmatists ignore this experimental refutation. The scientists take note of it and adapt their theories accordingly as advocated by Bacon, essentially the founder of the scientific method. Indeterminacy means that quantities are absolutely unknowable, and according to the dogmatists of Copenhagen, geometry is unknowable because general relativity is based on geometry. So they never succeeded in unifying general relativity and quantum mechanics. In ECE theory this unification is straightforward as just described; it is based on the tetrad postulate re-expressed as a wave equation. Anything that is claimed dogmatically to emanate from the fervent occult practices of indeterminacy can be obtained rationally and cooly from UFT13 without any fire or brimstone.

So indeterminacy was the first major casualty of ECE theory, other idols began to fall over, and the dogmatists with them. Everything has been thrown out of the window: U(1) gauge invariance, transverse vacuum radiation, the massless photon, the E(2) little group, the Einsteinian general relativity, the U(1) gauge invariance, the GWS electroweak theory, refuted completely in UFT 225, the SU(3) theory of quarks and gluons, quantum electrodynamics with its adjustable parameters such as virtual particles, the hocus pocus of renormalization and regularization, quantum chromodynamics, asymptotic freedom, quark confinement, approximate symmetry, string theory, superstring theory, multiple dimensions, nineteen adjustables, even more adjustables, yet more adjustables, dark matter, dark flow, big bang, black holes, interacting black holes, hundred billion dollar supercolliders, the whole lot, strange dreams leading to the Higgs boson, the murkiest idol of all.

Everything is cool and in the light of reason, everything is geometry.

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