2.7.2(6): Graphics of $\ddot{\theta}$ vs θ and $\ddot{r}$ vs. θ

1) The Graph of $\ddot{r}$ Versus θ

This is constructed from the following set of equations:

$$\ddot{r} = r\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) - \frac{MG}{r^2} \quad -(1)$$

$$\dot{\phi} = \frac{L_z}{mr^2\sin^2\theta}, \quad \dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_z^2}{\sin^2\theta}\right)^{1/2} \quad -(2)$$

$$r = \frac{\alpha}{1 + \epsilon\cos\beta} \quad -(3)$$

$$\cos\beta = \left(1 - \frac{L^2\cos^2\theta}{L^2 - L_z^2}\right)^{1/2} \quad -(4)$$

so $\ddot{r}$ can be expressed as a function of θ.

In two dimensions $\theta = \pi/2$ so there is no such function.

2) The Graph of $\ddot{\theta}$ Versus θ

This is constructed from:

$$\ddot{\theta} = \dot{\phi}^2 \sin\theta\cos\theta - 2\frac{\dot{r}\dot{\theta}}{r} \quad -(5)$$

2) where $\dot{\phi}$ and $\dot{\theta}$ are given by eq. (2), r by eqs. (3) and (4), and $\dot{r}$ by:

$$\dot{r} = \left(\frac{2}{m} \left(E - \frac{L^2}{2mr^2} + \frac{k}{r} \right) \right)^{1/2} \quad -(6)$$

Therefore $\ddot{\theta}$ can be plotted against θ.

$\frac{I}{II}$ 2 two dimensions there is no function of $\ddot{\theta}$ against θ because $\theta = \pi/2$ and $\ddot{\theta} = 0$.

Therefore these comparisons show that a three dimensional theory is far more richly structured than a two dimensional theory.
