272(8): The Graph of $\ddot{\phi}$ versus ϕ and θ.

In three dimensions:

$$\ddot{\phi} = -2\left(\dot{\theta}\frac{\cos\theta}{\sin\theta} + \frac{\dot{r}}{r}\right)\dot{\phi} \quad -(1)$$

where:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \quad -(2)$$

$$\cos\beta = \frac{\cos\phi}{\left(\cos^2\phi + \left(\frac{L_Z}{L}\right)^2\sin^2\phi\right)^{1/2}} \quad -(3)$$

$$\dot{r} = \left(\frac{2}{m}\left(E - \frac{L^2}{2mr^2} + \frac{k}{r}\right)\right)^{1/2} \quad -(4)$$

$$\dot{\phi} = \frac{L_Z}{mr^2\sin^2\theta} \quad -(5)$$

$$\dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2} \quad -(6)$$

$$\sin^2\theta = \left(\frac{L_Z}{L}\right)^2 + \left(1-\left(\frac{L_Z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L_Z}{L}\right)^2\sin^2\phi}\right) \quad -(7)$$

In two dimensions:

2)

$$\ddot{\phi} = -2\frac{\dot{r}}{r}\dot{\phi} \quad -(8)$$

where

$$\dot{\phi} = \frac{L_z}{mr^2} \quad -(9)$$

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(10)$$

$$\dot{r} = \left(\frac{2}{m}\left(E - \frac{L^2}{2mr^2} + \frac{k}{r}\right)\right)^{1/2} \quad -(11)$$

So in two dimensions $\ddot{\phi}$ is a function only of ϕ because $\theta = \pi/2$, $\dot{\theta} = 0$.

Type One

Express $\dot{\phi}$ in terms of θ using eq. (5) and $\dot{r}/r$ in terms of ϕ using eqs. (2) to (4), and graph $\ddot{\phi} = \ddot{\phi}(\phi, \theta)$. In this procedure express $\dot{\theta}$ in terms of θ using eq. (6).

Type Two

Express $\dot{\phi}$ in terms of ϕ using Eqs. (5) and (7), and express $\dot{r}/r$ in terms of θ using eqs. (2), (3) and (7). In this procedure express $\dot{\theta}$ in terms of θ using eq. (6) graph $\ddot{\phi} = \ddot{\phi}(\phi, \theta)$.

3)

Type Three

Repeat the procedure leading to type one but express $\dot{\theta}$ in terms of ϕ using eqs. (6) and (7).

Type Four

Repeat the procedure leading to type two but express $\dot{\theta}$ in terms of ϕ using eqs. (6) and (7).

Types Five, Six, Seven and Eight

Repeat the procedures leading to types one, two, ~~three~~ and four but express $\cos\theta$ / $\sin\theta$ in terms of ϕ using eq. (7) and

$$\cos^2\theta = 1 - \sin^2\theta \quad - (12)$$

There are many more permutations possible but all of these types may be expected to lead to orbital type structure.