$313(8)$ : Cecking and Carifying Note $31(6)$
Frm Eq. (14) onvards:

$$
\begin{aligned}
& {\left[D_{\rho},\left[D_{\mu}, D_{\mu} \cdot\right]\right] V^{k}=D_{\rho}\left(\left[D_{\mu}, D_{\mu}\right] V^{k}\right) } \\
= & D_{\rho}\left(R^{k} \lambda_{\mu \nu} V^{\lambda}-T_{\mu \nu}^{\lambda} D_{\lambda} V^{k r} D_{\mu}\right] D_{\rho} V^{k} \\
& -R^{k} \lambda_{\mu \nu} D_{\rho} V^{\lambda}+R_{\rho \mu \nu}^{\lambda} D_{\lambda} V^{k}+T_{\mu \nu}^{\lambda} D_{\lambda} D_{\rho} V^{k T} \\
= & D_{\rho} R^{k} \lambda_{\mu \nu} V^{\lambda}+R^{k} \lambda_{\mu \nu} D_{\rho} V^{\lambda} \\
& -D_{\rho} T_{\mu \nu}^{\lambda} D_{\lambda} V^{k}-T_{\mu \nu}^{\lambda} D_{\rho} D_{\lambda} V^{k} \\
& -R^{k} \lambda_{\mu \nu} D_{\rho} V^{\lambda}+R_{\rho \mu \nu}^{\lambda} D_{\lambda} V^{k}+T_{\mu \nu}^{\lambda} D_{\lambda} D_{\rho} V^{k} \\
= & D_{\rho} R^{k} \lambda_{\mu \mu} V^{\lambda}-D_{\rho} T_{\mu \nu}^{\lambda} D_{\lambda} V^{k}+R_{\rho \mu \nu}^{\lambda} D_{\lambda} V^{k} \\
& -T_{\mu \mu}^{\lambda^{k}}\left[D_{\rho \rho} D_{\lambda}\right] V^{k} \\
&
\end{aligned}
$$

The canple Jacosi idervity is terefar:

$$
\begin{aligned}
& \text { The cample }\left(D_{\rho},\left[D_{\mu}, D_{\sim}\right]\right]+\left[D_{\sim},\left[D_{\rho}, D_{\mu}\right]\right]+\left[D_{\mu},\left[D_{\sim}, D_{\rho}\right]\right] V^{k}
\end{aligned}
$$

$$
\begin{align*}
& \text { 2) }\left(R_{\rho \mu \nu}^{\lambda}+R_{\rho_{\mu \mu}}^{\lambda}+R_{\mu \nu}^{\lambda}-\left(D_{\rho} T_{\mu \mu}^{\lambda}+D_{\nu} T_{\rho \mu}^{\lambda}+D_{\mu} T_{\nu}^{\lambda}\right)\right) D_{\lambda} V^{k} \\
& -\left(T_{\mu \nu}^{\lambda}\left[D_{\rho}, D_{\lambda}\right]+T_{\rho \mu}^{\lambda}\left[D_{\nu}, D_{\lambda}\right]+T_{\nu \rho}^{\lambda}\left[D_{\mu} D_{\lambda}\right]\right) V^{k} \\
& :=0 \quad-(2) \tag{2}
\end{align*}
$$

Now use to Carton iterity:

$$
\begin{align*}
& \text { Now use te Cartan iserity: }  \tag{3}\\
& D_{\rho} T_{\mu \nu}^{\lambda}+D_{\nu} T_{\rho \mu}^{\lambda}+D_{\mu} T_{\nu \rho}^{\lambda}:=R_{\rho \mu \nu}^{\lambda}+R_{\nu \rho \mu}^{\lambda}+R_{\mu \nu}^{\lambda} \\
& -(3)
\end{align*}
$$

The Jacoli Cartan Evas (JCE) idesiyy:

$$
\begin{aligned}
& \left(\left[D_{\rho},\left[D_{\mu}, D_{\nu}\right]+\left[D_{\nu},\left[D_{\rho}, D_{\mu}\right]\right]+\left[D_{\mu},\left[D_{\nu}, D_{\rho}\right]\right]\right) V^{k}\right. \\
& :=\left(T_{\mu \nu}^{\lambda}\left[D_{\rho}, D_{\lambda}\right]+T_{\rho \mu}^{\lambda}\left[D_{\nu}, D_{\lambda}\right]+T_{\nu}^{\lambda}\left[D_{\mu}, D_{\lambda}\right]\right) V^{k} \\
& \begin{aligned}
& =\left(D_{\rho} R_{\lambda \mu}^{\prime \prime}+D_{\nu} R_{\lambda \mu \mu}^{\prime}+D_{\mu} R_{\lambda \mu}^{k}\right) V_{-(4)}^{\lambda}
\end{aligned} \\
& \text { ILClis idenity: } \\
& \left(T_{\mu \mu}^{\lambda_{\mu}}\left[D_{\rho}, D_{\lambda}\right]+T_{\rho \mu}^{\lambda}\left[D_{0}, D_{\lambda}\right]+T_{\rho \rho}^{\lambda}\left[D_{\mu}, D_{\lambda}\right]\right) V^{\kappa}
\end{aligned}
$$

3) 

$$
\begin{align*}
& =\left(T_{\mu \nu}^{\lambda} R_{\alpha \rho \lambda}^{k}+T_{\rho \mu}^{\lambda} R_{\alpha \nu \lambda}^{k_{i}}+T_{\nu \rho}^{\lambda} R_{\alpha \mu \lambda}^{k}\right) V^{\alpha} \\
& -\left(T_{\mu \nu}^{\lambda} T_{\rho \lambda}{ }^{\alpha}+T_{\rho \mu}^{\lambda} T_{\nu \lambda}^{\alpha}+T_{\nu \rho}^{\lambda} T_{\mu \lambda}^{\alpha}\right) D_{\alpha} V^{k}  \tag{5}\\
& -(5)
\end{align*}
$$

wetce fist Evano idensity:

$$
\begin{align*}
& \text { Qe fist Evano idewity: }  \tag{6}\\
& T_{\mu \nu}^{\lambda} T_{\rho}^{\alpha}+T_{\rho \mu}^{\lambda} T_{\rightarrow \lambda}^{\alpha}+T_{j}^{\lambda} T_{\mu \lambda}^{\alpha}:=0 \\
& -(6)
\end{align*}
$$

S. the Jacobi Cartar Furano iderity is:

$$
\begin{aligned}
& \left(\left[D_{\rho},\left[D_{\mu}, D\right]+\left[D_{\nu},\left[D_{\rho}, D_{\mu}\right]\right]+\left[D_{\mu},\left[D_{\nu}, D_{\rho}\right]\right]^{k} V^{k}\right.\right. \\
& =\left(D_{\rho} R^{k} \lambda_{\mu \nu}+D_{\nu} R^{k} \lambda_{\rho \mu}+D_{\mu} R^{k} \lambda_{\nu \rho}\right) V^{\lambda} \\
& +\left(T_{\mu \nu}^{\lambda} R_{\alpha \rho \lambda}^{k}+T_{\rho \mu}^{\lambda} R_{\alpha \nu \lambda}^{k}+T_{\nu_{\rho}}^{\lambda} R_{\alpha \mu \lambda}^{k}\right) V^{\alpha} \\
& =\left(D_{\rho} R^{k} \lambda_{\mu \nu}+D_{\mu} R^{k} \lambda_{\rho_{\mu}}+D_{\mu} R^{k} \lambda_{\alpha_{\mu}}\right. \\
& \left.+T_{\mu^{\alpha}}^{\alpha} R_{\lambda \rho \alpha}^{k}+T_{p \mu}^{\alpha} R_{\lambda \omega \alpha}^{k}+T_{\mu \rho}^{\alpha} R_{\lambda \mu \alpha}^{k}\right) V^{\lambda} \\
& -(7)
\end{aligned}
$$

4) The arimal 1902 Biach idenity is:

$$
D_{\rho} R^{k} \lambda_{\mu \nu}+D_{\nu} R^{K} \lambda_{\mu}+D_{\mu} R_{\lambda \mu \rho}=0-(8)
$$

and $Q_{j}$ is no longer tme in te presarce of tacsion.
Th Jacali ikewity (2) give Jot ct Cantar: dentiky (3) and te forst Evano iderity (6), as well a th SCE idenity (7).

