435(4): Free Particle Time Dependent Schroedinger Equation

This is:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial r^2} \quad -(1)$$

where

$$\psi = e^{-i\omega t} e^{i\kappa r} \quad -(2)$$

is the wave function. Eq. (1) is:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi = H\psi \quad -(3)$$

So the energy levels of the free particle are:

$$E = \langle E \rangle = i\hbar \int \psi^* \frac{\partial \psi}{\partial t}\, d\tau \quad -(4)$$

$$= \hbar\omega \int \psi^* \psi \, d\tau = \hbar\omega$$

These are equal to:

$$E = -\frac{\hbar^2}{2m}\int \psi^* \frac{\partial^2 \psi}{\partial r^2}\, d\tau$$

$$= \frac{\hbar^2 \kappa^2}{2m}\int \psi^* \psi \, d\tau \quad -(5)$$

$$= \frac{\hbar^2 \kappa^2}{2m} = \frac{p^2}{2m}$$

So

$$\boxed{E = \hbar\omega = \frac{\hbar^2\kappa^2}{2m}} \quad -(6)$$

These are kinetic energy levels of a free particle.

In the frame of m theory the time dependent Schroedinger equation is:

$$\frac{i\hbar\, \partial \phi_1}{\partial t_1} = -\frac{\hbar^2}{2m}\frac{\partial^2 \phi_1}{\partial r_1^2} \quad -(7)$$

where:

$$\phi_1 = \exp(-i\omega_1 t_1)\exp(i\kappa_1 r_1) \quad -(8)$$

However, the phase is invariant under frame transformation, so:

$$\omega_1 t_1 - \kappa_1 r_1 = \omega t - \kappa r \quad -(9)$$

So:

$$\phi_1 = \phi \quad -(10)$$

The time dependent Schroedinger equation is therefore:

$$\frac{i\hbar}{m(r)^{1/2}}\frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \phi}{\partial r_1^2}. \quad -(11)$$

So the energy levels of the free particle in the frame are:

$$E_1 = \hbar\omega \int \phi^* \frac{1}{m(r)^{1/2}} \phi \, d\tau \quad -(12)$$

$$= \frac{\hbar^2 \kappa_1^2}{2m}$$

$$= \frac{p^2}{2m(r)\, m}$$

The transformation to m space produces a shift in energy