435(5): Brief Review of Fundamental Concepts

In 4FT415 the infinitesimal line element:

$$ds^2 = m(r)c^2dt^2 - \frac{dr^2}{m(r)} - r^2d\phi^2 \quad -(1)$$

was introduced and used to produce the result:

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(2)$$

The Minkowski metric is:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\phi^2 \quad -(3)$$

$$= c^2d\tau^2$$

So:

$$c^2\left(\frac{d\tau}{dt}\right)^2 = c^2 - \left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(4)$$

and

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{1}{c^2}\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2\right) \quad -(5)$$

where

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(6)$$

The Lorentz factor is therefore:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(7)$$

Using eq. (1) on the other hand:

$$\left(\frac{d\tau}{dt}\right)^2 = m(r) - \frac{v^2}{c^2} \quad -(8)$$

where

$$v^2 = \frac{1}{m(r)}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(9)$$

On the other hand, if:

$$\underline{r} = \frac{r}{m(r)^{1/2}}\,\underline{e}_r \quad -(10)$$

2) then:

$$\underline{\dot{r}} = \frac{d}{dt}\left(\frac{r}{n(r)^{1/2}}\underline{e}_r\right) \quad -(11)$$

$$= \frac{1}{n(r)^{1/2}}\frac{d}{dt}\left(r\underline{e}_r\right)$$

$$= \frac{1}{n(r)^{1/2}}\left(\dot{r}\underline{e}_r + r\underline{\dot{e}}_r\right)$$

$$= \frac{1}{n(r)^{1/2}}\left(\dot{r}\underline{e}_r + r\dot{\phi}\underline{e}_\phi\right)$$

Therefore:

$$v^2 = \underline{\dot{r}} \cdot \underline{\dot{r}} = \frac{1}{n(r)}\left(\dot{r}^2 + r^2\dot{\phi}^2\right) \quad -(12)$$

Note that Eqs. (9) and (12) are not ~~self~~ self consistent.

In UFT 416 it was found that the internally self consistent infinitesimal line element is the Minkowski line element in n space:

$$\boxed{ds^2 = c^2 dt_1^2 - dr_1^2 - r_1^2 d\phi^2} \quad -(13)$$

which is an example of the general metric:

$$ds^2 = g_{aa}(a,b)\,da^2 + g_{ab}(a,b)(da\,db + db\,da) \quad -(14)$$
$$+ g_{bb}(a,b)\,db^2 + r^2(a,b)\,d\phi^2$$

Eq. (13) can be written as:

$$ds^2 = c^2 d\tau^2 = n(r)c^2 dt^2 - dr_1^2 - r_1^2 d\phi^2 \quad -(15)$$

with

$$r_1 = \frac{r}{n(r)^{1/2}} \quad -(16)$$

It follows that:

$$\left(\frac{d\tau}{dt}\right)^2 = m(r) - \frac{1}{c^2}\left(\left(\frac{dr_1}{dt}\right)^2 + r_1^2\left(\frac{d\phi}{dt}\right)^2\right) \quad -(17)$$

Defining:

$$v_1^2 = \left(\frac{dr_1}{dt}\right)^2 + r_1^2\left(\frac{d\phi}{dt}\right)^2 \quad -(18)$$

it follows that:

$$\gamma_1 = \left(m(r_1) - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(19)$$

$$= \left(m(r_1) - \frac{v^2}{m(r_1)c^2}\right)^{-1/2}$$

The geodesic method was then used to produce the following self consistent results. The relativistic total energy is:

$$E_1 = m(r_1)\gamma_1 mc^2 \quad -(20)$$

The relativistic momentum is:

$$p_1 = \frac{1}{m(r_1)^{1/2}} m\frac{dr}{dt}. \quad -(21)$$

The relativistic angular momentum is the constant of motion:

$$L_1 = \frac{\gamma_1 m r^2}{m(r_1)}\frac{d\phi}{dt} \quad -(22)$$

The gravitational lagrangian is:

$$\mathcal{L} = -mc^2\left(m(r_1) - \frac{v_1^2}{c^2}\right)^{1/2} + \frac{mMG}{r_1} \quad -(23)$$

producing the force:

$$\underline{F_1} = \frac{dp_1}{dt} = \left(-\frac{mc^2}{2}\gamma_1\frac{dm(r_1)}{dr_1} - \frac{mMG}{r_1^2}\right)\underline{e}_r \quad -(24)$$

+) The force due to the m space itself is:

$$\underline{F}(\text{vac}) = -\frac{mc^2}{2}\gamma_1 \frac{dm(r_1)}{dr_1} \quad -(25)$$

$$= mc^2 \left(\frac{\gamma_1 m(r)^{3/2} \frac{dm(r)}{dr}}{r\frac{dm(r)}{dr} - 2m(r)} \right) \underline{e}_r$$

and this can become infinite.

These concepts are rigorously self consistent and were later shown to be consistent with Hamiltonian dynamics.

The fundamental replacements are:

$$\boxed{\begin{aligned} r \rightarrow r_1 &= \frac{r}{m(r)^{1/2}} && -(26) \\ t \rightarrow t_1 &= m(r)^{1/2} t && -(27) \end{aligned}}$$

and

In quantum mechanics these lead to shifts and splittings due to the m space itself.

The usual Schroedinger quantization is:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \quad -(28)$$

and

$$-i\hbar \underline{\nabla} \psi = \underline{p}\psi \quad -(29)$$

so the time dependent Schroedinger quantization is:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi = H\psi \quad -(30)$$

In m space these equations become:

5) Consider the free particle wave function:

$$\psi(r) = e^{ikr} \quad -(31)$$

In the n space defined by Eq. (13) this becomes:

$$\psi(r) = \exp\left(\frac{ikr}{n(r)^{1/2}}\right) \quad -(32)$$

and the wave function $\exp(-i\omega t)$ becomes:

$$\psi(t) = \exp\left(-i\omega n^{1/2}(r)t\right) \quad -(33)$$

So

$$\psi(t, r) = \psi(t)\psi(r) \quad -(34)$$

$$= \exp\left(-i\omega n^{1/2}(r)t\right)\exp\left(\frac{ikr}{n(r)^{1/2}}\right)$$

so the energy levels from the time dependent Schroedinger equation are:

$$\boxed{E = n(r)^{1/2}\hbar\omega \int \psi^* \frac{1}{n(r)^{1/2}}\psi \, d\tau} \quad -(35)$$

Eq. (35) is valid for all wave functions ψ, and is the Planck quantization in n space.