1)

O LEVEL LESSON THREE

From lesson one:

$$D_{\mu\nu}\left(V^{\rho}\right) = -\Gamma^{\lambda}_{\mu\nu}\left(D_{\lambda} V^{\rho}\right) + \dots \qquad (1)$$

where we denote:

$$D_{\mu\nu} := \left[D_{\mu}, D_{\nu}\right] \qquad (2)$$

Focus in on the indices μ and ν, and for convenience write

$$\boxed{D_{\mu\nu} \rightarrow -\Gamma^{\lambda}_{\mu\nu}} \qquad (3).$$

Everyone agrees that

$$D_{\mu\nu} = -D_{\nu\mu} \qquad (4)$$

and that when

$$\mu = \nu \qquad (5)$$

$$D_{\mu\nu} = 0 \qquad (6)$$

So from eq (3):

$$\boxed{\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}} \qquad (7)$$

and when μ and ν are the same:

$$\Gamma^{\lambda}_{\mu\nu} = 0. \qquad (8)$$

2) Eq. (7) is the logically correct ECE interpretation.

The <u>incorrect</u> standard model interpretation is:

$$D_{\mu\nu}(V^\rho) = -T^\lambda_{\mu\nu}(D_\lambda V^\rho) + \dots \quad (9)$$

i.e.

$$\boxed{D_{\mu\nu} \rightarrow T^\lambda_{\mu\nu}.} \quad (10)$$

Here:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \quad (11)$$

so

$$\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = -\left(\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}\right) \quad (12)$$

or

$$T_{\mu\nu} = -T_{\nu\mu} \quad (13)$$

Eq. (12) can be written as:

$$\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\mu} = \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\mu\nu} \quad (14)$$

i.e.

$$\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\mu} = \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\mu} \quad (15)$$

In the standard model, the symmetric connection is

$$\Gamma^\lambda_{\mu\nu}(S) = \frac{1}{2}\left(\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\mu}\right) \quad (16)$$

so <u>the</u> standard model uses:

3) $$\Gamma^{\lambda}_{\mu\nu}(S) = \Gamma^{\lambda}_{\nu\mu}(S) \quad -(17)$$

However, from eq. (7) it is seen that eq. (17) means:

$$\boxed{\Gamma^{\lambda}_{\mu\nu}(S) = 0} \quad -(18)$$

The symmetric connection is zero.

The antisymmetric connection is:

$$\Gamma^{\lambda}_{\mu\nu}(A) = \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right) \quad -(19)$$

and from eq. (7):

$$\boxed{\Gamma^{\lambda}_{\mu\nu}(A) = \Gamma^{\lambda}_{\mu\nu} \neq 0} \quad -(20)$$

From a well known theorem of matrices, it is always possible to write:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}(S) + \Gamma^{\lambda}_{\mu\nu}(A) \quad -(21)$$

Therefore from eqs. (18) and (20):

$$\boxed{\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}(A)} \quad -(22)$$