ECE Engineering Model

The Basis for Electromagnetic and Mechanical Applications

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ECE Field Equations

- Field equations in tensor form

$$\partial_\mu F^{a\mu\nu} = \frac{1}{\varepsilon_0} j^{a\nu}$$

$$\partial_\mu \tilde{F}^{a\mu\nu} = \mu_0 J^{a\nu}$$

- With
  - F: electromagnetic field tensor, $\tilde{F}$ its Hodge dual, see later
  - J: charge current density
  - j: „homogeneous current density“, „magnetic current“
  - a: polarization index
  - μ,v: indexes of spacetime (t,x,y,z)

Properties of Field Equations

- J is not necessarily external current, is defined by spacetime properties completely
- j only occurs if electromagnetism is influenced by gravitation, otherwise =0
- Polarization index „a“ can be omitted if tangent space is defined equal to space of base manifold (assumed from now on)

Electromagnetic Field Tensor

- F and F~ are antisymmetric tensors, related to vector components of electromagnetic fields (polarization index omitted)

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{pmatrix}$$

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & -cB^1 & -cB^2 & -cB^3 \\ cB^1 & 0 & -E^3 & E^2 \\ cB^2 & E^3 & 0 & -E^1 \\ cB^3 & -E^2 & E^1 & 0 \end{pmatrix}$$

Potential with polarization directions

- Potential matrix:

$$\begin{pmatrix} \Phi^{(0)} & \Phi^{(1)} & \Phi^{(2)} & \Phi^{(3)} \\ 0 & A_1^{(1)} & A_1^{(2)} & A_1^{(3)} \\ 0 & A_2^{(1)} & A_2^{(2)} & A_2^{(3)} \\ 0 & A_3^{(1)} & A_3^{(2)} & A_3^{(3)} \end{pmatrix}$$

- Polarization vectors:

$$\mathbf{A}^{(1)} = \begin{pmatrix} A_1^{(1)} \\ A_2^{(1)} \\ A_3^{(1)} \end{pmatrix}, \quad \mathbf{A}^{(2)} = \begin{pmatrix} A_1^{(2)} \\ A_2^{(1)} \\ A_3^{(2)} \end{pmatrix}, \quad \mathbf{A}^{(3)} = \begin{pmatrix} A_1^{(3)} \\ A_2^{(3)} \\ A_3^{(3)} \end{pmatrix}$$

ECE Field Equations – Vector Form

$$\nabla \cdot \mathbf{B}^a = \mu_0 \rho_{eh}{}^a = \rho_{eh}{}^{a\prime} = 0$$ Gauss Law

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}_{eh}{}^a = \mathbf{j}_{eh}{}^{a\prime} = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E}^a = \frac{\rho_e{}^a}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}_e{}^a$$ Ampère - Maxwell Law

„Material“ Equations

$$\mathbf{D}^a = \varepsilon_r \varepsilon_0 \mathbf{E}^a$$ Dielectric Displacement

$$\mathbf{B}^a = \mu_r \mu_0 \mathbf{H}^a$$ Magnetic Induction

Physical Units

$$[\mathbf{E}] = \frac{V}{m}$$

$$[\mathbf{B}] = T = \frac{V \cdot s}{m^2} = \frac{N}{A \cdot m}$$

$$[\mathbf{D}] = \frac{C}{m^2}, \quad [\mathbf{H}] = \frac{A}{m}$$

$$[\Phi] = V$$

$$[\mathbf{A}] = \frac{Vs}{m}$$

$$[\omega_0] = \frac{1}{s}$$

$$[\boldsymbol{\omega}] = \frac{1}{m}$$

Charge Density/Current

$$[\rho_e] = C / m^3$$

$$[\mathbf{J}_e] = A / m^2 = C / (m^2 s)$$

„Magnetic“ Density/Current

$$[\rho_{eh}] = \frac{A}{m^2} \qquad [\rho_{eh}'] = \frac{Vs}{m^3}$$

$$[\mathbf{j}_{eh}] = \frac{A}{ms} \qquad [\mathbf{j}_{eh}'] = \frac{V}{m^2}$$

Field-Potential Relations I
Full Equation Set

$$\mathbf{E}^a = -\nabla\Phi^a - \frac{\partial \mathbf{A}^a}{\partial t} - \omega_0{}^a{}_b \mathbf{A}^b + \boldsymbol{\omega}^a{}_b \Phi^b$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b$$

Potentials and Spin Connections

$\mathbf{A}^a$: Vector potential
Φ^a: scalar potential
$\boldsymbol{\omega}^a{}_b$: Vector spin connection
$\omega_0{}^a{}_b$: Scalar spin connection

Please observe the Einstein summation convention!

ECE Field Equations in Terms of Potential I

Gauss Law :

$$\nabla \cdot (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b) = 0$$

Faraday Law of Induction :

$$-\nabla \times (\omega_0{}^a{}_b \mathbf{A}^b) + \nabla \times (\boldsymbol{\omega}^a{}_b \Phi^b) - \frac{\partial (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b)}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}^a}{\partial t} - \Delta \Phi^a - \nabla \cdot (\omega_0{}^a{}_b \mathbf{A}^b) + \nabla \cdot (\boldsymbol{\omega}^a{}_b \Phi^b) = \frac{\rho_e{}^a}{\varepsilon_0}$$

Ampère - Maxwell Law :

$$\nabla (\nabla \cdot \mathbf{A}^a) - \Delta \mathbf{A}^a - \nabla \times (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b)$$

$$+ \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{A}^a}{\partial t^2} + \frac{\partial (\omega_0{}^a{}_b \mathbf{A}^b)}{\partial t} + \nabla \frac{\partial \Phi^a}{\partial t} - \frac{\partial (\boldsymbol{\omega}^a{}_b \Phi^b)}{\partial t} \right) = \mu_0 \mathbf{J}_e{}^a$$

Antisymmetry Conditions of ECE Field Equations I

Electric antisymmetry constraints:

$$\nabla \Phi^a - \frac{\partial \mathbf{A}^a}{\partial t} - \omega_0{}^a{}_b \mathbf{A}^b - \boldsymbol{\omega}^a{}_b \Phi^b = 0$$

Magnetic antisymmetry constraints:

$$\frac{\partial A^a{}_3}{\partial x_2} + \frac{\partial A^a{}_2}{\partial x_3} + \omega^a{}_{b,2} A^b{}_3 + \omega^a{}_{b,3} A^b{}_2 = 0$$

$$\frac{\partial A^a{}_3}{\partial x_1} + \frac{\partial A^a{}_1}{\partial x_3} + \omega^a{}_{b,1} A^b{}_3 + \omega^a{}_{b,3} A^b{}_1 = 0$$

$$\frac{\partial A^a{}_2}{\partial x_1} + \frac{\partial A^a{}_1}{\partial x_2} + \omega^a{}_{b,1} A^b{}_2 + \omega^a{}_{b,2} A^b{}_1 = 0$$

Or simplified Lindstrom constraint:

$$\nabla \times \mathbf{A}^a + \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = 0$$

Field-Potential Relations II
One Polarization only

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} + \boldsymbol{\omega}\Phi$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}$$

Potentials and Spin Connections

$\mathbf{A}$: Vector potential
Φ: scalar potential
$\boldsymbol{\omega}$: Vector spin connection
ω_0: Scalar spin connection

ECE Field Equations in Terms of Potential II

Gauss Law :

$$\nabla \cdot (\boldsymbol{\omega} \times \mathbf{A}) = 0$$

Faraday Law of Induction :

$$-\nabla \times (\omega_0 \mathbf{A}) + \nabla \times (\boldsymbol{\omega}\Phi) - \frac{\partial(\boldsymbol{\omega} \times \mathbf{A})}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \Delta\Phi - \nabla \cdot (\omega_0 \mathbf{A}) + \nabla \cdot (\boldsymbol{\omega}\Phi) = \frac{\rho_e}{\varepsilon_0}$$

Ampère - Maxwell Law :

$$\nabla(\nabla \cdot \mathbf{A}) - \Delta\mathbf{A} - \nabla \times (\boldsymbol{\omega} \times \mathbf{A})$$

$$+\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{\partial(\omega_0 \mathbf{A})}{\partial t} + \nabla\frac{\partial\Phi}{\partial t} - \frac{\partial(\boldsymbol{\omega}\Phi)}{\partial t}\right) = \mu_0 \mathbf{J}_e$$

ECE Field Equations in Terms of Potential with “cold“ currents II

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \Delta \Phi = \frac{\rho_{e0}}{\varepsilon_0}$$

$$-\nabla \cdot (\omega_0 \mathbf{A}) + \nabla \cdot (\boldsymbol{\omega} \Phi) = \frac{\rho_{e1}}{\varepsilon_0}$$

Ampère - Maxwell Law :

$$\nabla(\nabla \cdot \mathbf{A}) - \Delta \mathbf{A} + \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla \frac{\partial \Phi}{\partial t}\right) = \mu_0 \mathbf{J}_{e0}$$

$$-\nabla \times (\boldsymbol{\omega} \times \mathbf{A}) + \frac{1}{c^2}\left(\frac{\partial(\omega_0 \mathbf{A})}{\partial t} - \frac{\partial(\boldsymbol{\omega}\Phi)}{\partial t}\right) = \mu_0 \mathbf{J}_{e1}$$

ρ_{e0}, J_{e0}: normal charge density and current

ρ_{e1}, J_{e1}: “cold“ charge density and current

Antisymmetry Conditions of ECE Field Equations II

Electric antisymmetry constraints:

$$\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} - \boldsymbol{\omega}\Phi = 0$$

Magnetic antisymmetry constraints:

$$\frac{\partial A_3}{\partial x_2} + \frac{\partial A_2}{\partial x_3} + \omega_2 A_3 + \omega_3 A_2 = 0$$

$$\frac{\partial A_3}{\partial x_1} + \frac{\partial A_1}{\partial x_3} + \omega_1 A_3 + \omega_3 A_1 = 0$$

$$\frac{\partial A_2}{\partial x_1} + \frac{\partial A_1}{\partial x_2} + \omega_1 A_2 + \omega_2 A_1 = 0$$

or:

$$\nabla \times \mathbf{A} + \boldsymbol{\omega} \times \mathbf{A} = 0$$

All these relations appear in addition to the ECE field equations and are constraints of them. They replace Lorenz Gauge invariance and can be used to derive special properties.

Field-Potential Relations III
Linearized Equations

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\omega}_E$$

$$\mathbf{B} = \nabla \times \mathbf{A} + \boldsymbol{\omega}_B$$

Potentials and Spin Connections

$\mathbf{A}$: Vector potential

Φ: scalar potential

$\boldsymbol{\omega}_E$: Vector spin connection of electric field

$\boldsymbol{\omega}_B$: Vector spin connection of magnetic field

ECE Field Equations in Terms of Potential III

Gauss Law :

$$\nabla \cdot \boldsymbol{\omega}_B = 0$$

Faraday Law of Induction :

$$\nabla \times \boldsymbol{\omega}_E + \frac{\partial \boldsymbol{\omega}_B}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \Delta \Phi + \nabla \cdot \boldsymbol{\omega}_E = \frac{\rho_e}{\varepsilon_0}$$

Ampère - Maxwell Law :

$$\nabla(\nabla \cdot \mathbf{A}) - \Delta \mathbf{A} + \nabla \times \boldsymbol{\omega}_B$$

$$+ \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla \frac{\partial \Phi}{\partial t} - \frac{\partial \boldsymbol{\omega}_E}{\partial t} \right) = \mu_0 \mathbf{J}_e$$

Antisymmetry Conditions of ECE Field Equations III

Define additional vectors $\boldsymbol{\omega}_{E1}$, $\boldsymbol{\omega}_{E2}$, $\boldsymbol{\omega}_{B1}$, $\boldsymbol{\omega}_{B2}$:

$$\boldsymbol{\omega}_E = -\left(\boldsymbol{\omega}_{E1} - \boldsymbol{\omega}_{E2}\right)$$

$$\boldsymbol{\omega}_B = -\left(\boldsymbol{\omega}_{B1} - \boldsymbol{\omega}_{B2}\right)$$

Electric antisymmetry constraints:

$$\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\omega}_{E1} + \boldsymbol{\omega}_{E2} = 0$$

Magnetic antisymmetry constraints:

$$\begin{pmatrix} \dfrac{\partial A_3}{\partial x_2} + \dfrac{\partial A_2}{\partial x_3} \\ \dfrac{\partial A_1}{\partial x_3} + \dfrac{\partial A_3}{\partial x_1} \\ \dfrac{\partial A_2}{\partial x_1} + \dfrac{\partial A_1}{\partial x_2} \end{pmatrix} + \boldsymbol{\omega}_{B1} + \boldsymbol{\omega}_{B2} = 0$$

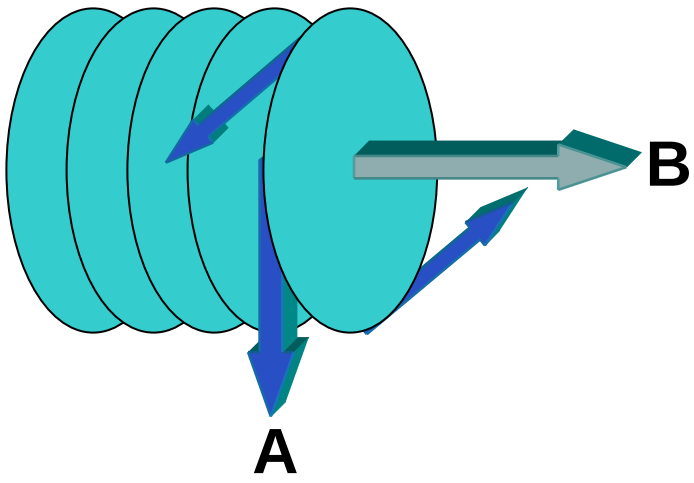
Properties of ECE Equations

- The ECE equations in potential representation define a well-defined equation system (8 equations with 8 unknows)
- There is much more structure in ECE than in standard theory (Maxwell-Heaviside)
- There is no gauge freedom in ECE theory
- In potential representation, the Gauss and Faraday law do not make sense in standard theory (see red fields)
- Resonance structures (self-enforcing oscillations) are possible in Coulomb and Ampère-Maxwell law

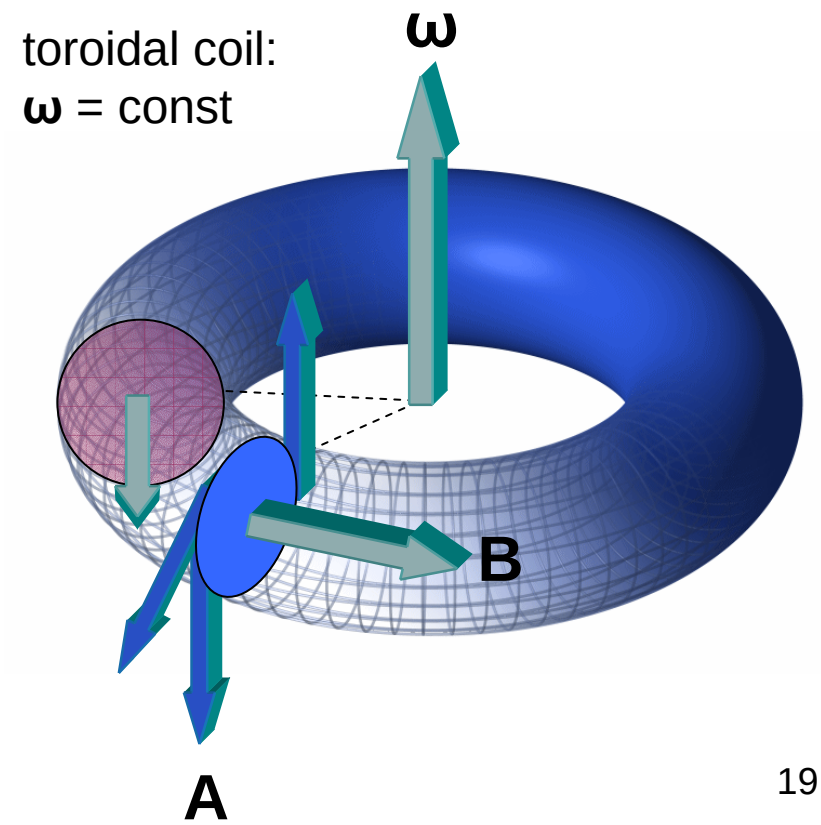
Examples of Vector Spin Connection

Vector spin connection **ω** represents rotation of plane of **A** potential

linear coil:
ω = 0



toroidal coil:
ω = const



ECE Field Equations of Dynamics

$$\nabla \cdot \mathbf{h} = 4\pi G \rho_{mh} = 0$$ (Equivalent of Gauss Law)

$$\nabla \times \mathbf{g} + \frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} = \frac{4\pi G}{c}\mathbf{j}_{mh} = 0$$ Gravito - magnetic Law

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m$$ Newton's Law (Poisson equation)

$$\nabla \times \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi G}{c}\mathbf{J}_m$$ (Equivalent of Ampère - Maxwell Law)

Only Newton‘s Law is known in the standard model.

ECE Field Equations of Dynamics Alternative Form with $\mathbf{\Omega}$

$$\nabla \cdot \mathbf{\Omega} = \frac{4\pi G}{c} \rho_{mh} = 0 \qquad \text{(Equivalent of Gauss Law)}$$

$$\nabla \times \mathbf{g} + \frac{\partial \mathbf{\Omega}}{\partial t} = \frac{4\pi G}{c} \mathbf{j}_{mh} = 0 \qquad \text{Gravito-magnetic Law}$$

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m \qquad \text{Newton's Law (Poisson equation)}$$

$$\nabla \times \mathbf{\Omega} - \frac{1}{c^2} \frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi G}{c^2} \mathbf{J}_m \qquad \text{(Equivalent of Ampère-Maxwell Law)}$$

Alternative gravito-magnetic field: $\mathbf{\Omega} = \dfrac{\mathbf{h}}{c}$

Only Newton‘s Law is known in the standard model.

Fields, Currents and Constants

Fields and Currents

g: gravity acceleration
ρ_m: mass density
J_m: mass current

Ω, **h**: gravito-magnetic field
ρ_{mh}: gravito-magn. mass density
j_{mh}: gravito-magn. mass current

Constants

G: Newton‘s gravitational constant
c: vacuum speed of light, required for correct physical units

Force Equations

$\mathbf{F} = m\mathbf{g}$	Newtonian Force Law
$\mathbf{F} = E_0\mathbf{T}$	Torsional Force Law
$\mathbf{F}_{\mathrm{L}} = mc\mathbf{v}\times\mathbf{h}$	Lorentz Force Law
$\mathbf{M} = \dfrac{\partial\mathbf{L}}{\partial t} - \boldsymbol{\Theta}\times\mathbf{L}$	Torque Law

Physical quantities and units

F [N]	Force
M [Nm]	Torque
T [1/m]	Torsion
g, h [m/s^2]	Acceleration
m [kg]	Mass
v [m/s]	Mass velocity
E_0=mc^2 [J]	Rest energy
$\boldsymbol{\Theta}$ [1/s]	Rotation axis vector
L [Nms]	Angular momentum

Field-Potential Relations

$$\mathbf{g} = -\frac{\partial \mathbf{Q}}{\partial t} - \nabla\Phi - \omega_0 \mathbf{Q} + \boldsymbol{\omega}\Phi$$

$$\boldsymbol{\Omega} = \frac{\mathbf{h}}{c} = \nabla \times \mathbf{Q} - \boldsymbol{\omega} \times \mathbf{Q}$$

Potentials and Spin Connections

$\mathbf{Q}$=c$\mathbf{q}$: Vector potential
Φ: Scalar potential
$\boldsymbol{\omega}$: Vector spin connection
ω_0: Scalar spin connection

Physical Units

Fields

$$[\mathbf{g}] = [\mathbf{h}] = \frac{m}{s^2}$$

$$[\mathbf{\Omega}] = \frac{1}{s}$$

Potentials

$$[\Phi] = \frac{m^2}{s^2}$$

$$[\mathbf{Q}] = \frac{m}{s}$$

Spin Connections

$$[\omega_0] = \frac{1}{s}$$

$$[\boldsymbol{\omega}] = \frac{1}{m}$$

Constants

$$[G] = \frac{m^3}{kg\, s^2}$$

Mass Density/Current

$$[\rho_m] = \frac{kg}{m^3}$$

$$[J_m] = \frac{kg}{m^2 s}$$

„Gravito-magnetic“ Density/Current

$$[\rho_{mh}] = \frac{kg}{m^3}$$

$$[j_m] = \frac{kg}{m^2 s}$$

Antisymmetry Conditions of ECE Field Equations of Dynamics

Relations for

classical and ECE Potenitals :

$$\nabla \Phi = \frac{\partial \mathbf{Q}}{\partial t}$$

$$\frac{\partial Q_1}{\partial x_2} = -\frac{\partial Q_2}{\partial x_1}$$

$$\frac{\partial Q_1}{\partial x_3} = -\frac{\partial Q_3}{\partial x_1}$$

$$\frac{\partial Q_2}{\partial x_3} = -\frac{\partial Q_3}{\partial x_2}$$

Relations for

spin connections :

$$\omega_0 \mathbf{Q} = -\boldsymbol{\omega} \Phi$$

$$\omega_1 Q_2 = -\omega_2 Q_1$$

$$\omega_1 Q_3 = -\omega_3 Q_1$$

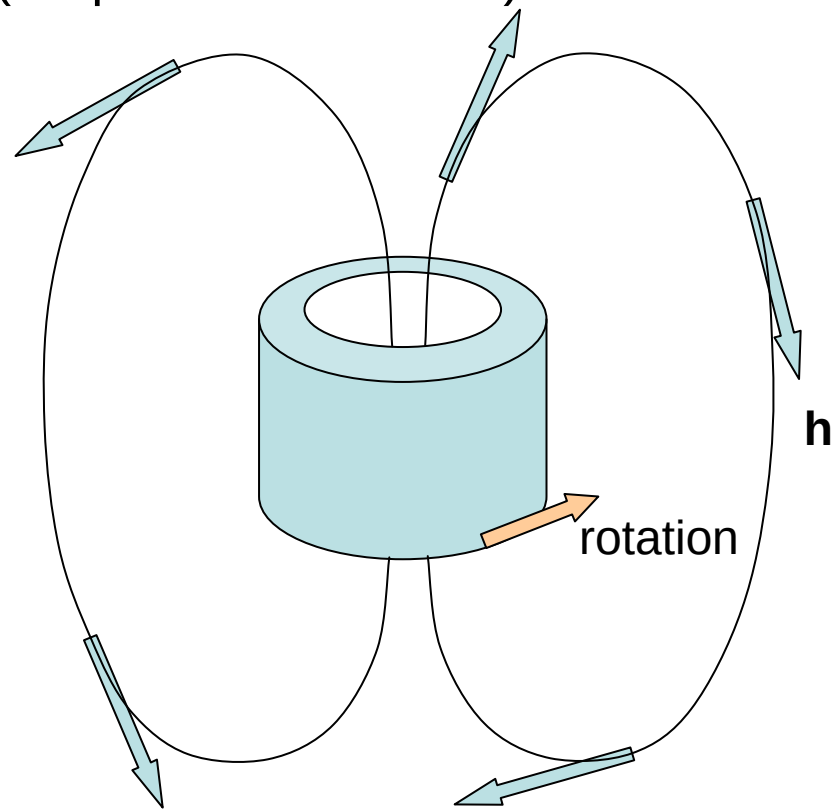
$$\omega_2 Q_3 = -\omega_3 Q_2$$

Properties of ECE Equations of Dynamics

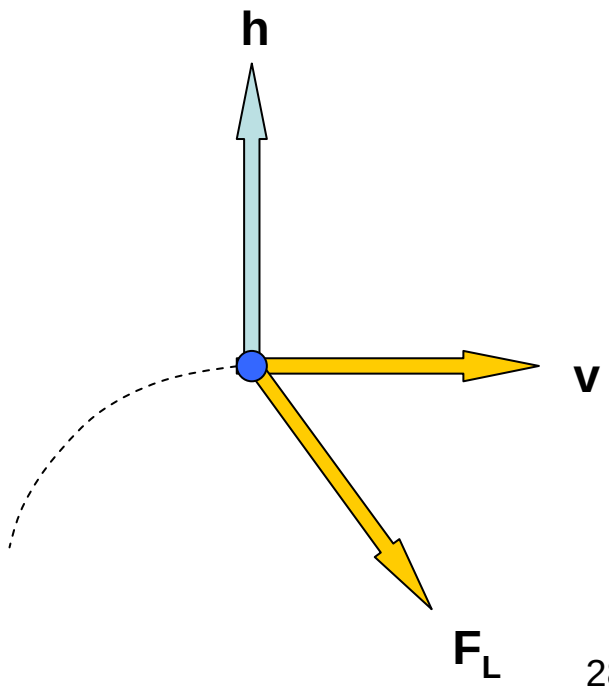
- Fully analogous to electrodynamic case
- Only the Newton law is known in classical mechanics
- Gravito-magnetic law is known experimentally (ESA experiment)
- There are two acceleration fields **g** and **h**, but only **g** is known today
- **h** is an angular momentum field and measured in m/s^2 (units chosen the same as for **g**)
- Mechanical spin connection resonance is possible as in electromagnetic case
- Gravito-magnetic current occurs only in case of coupling between translational and rotational motion

Examples of ECE Dynamics

Realisation of gravito-magnetic field **h** by a rotating mass cylinder (Ampere-Maxwell law)



Detection of **h** field by mechanical Lorentz force $\mathbf{F}_L$
v: velocity of mass m



Polarization and Magnetization

Electromagnetism

P: Polarization
M: Magnetization

$$D = \varepsilon_0 E + P$$

$$[P] = \frac{C}{m^2}$$

$$B = \mu_0 (H + M)$$

$$[M] = \frac{A}{m}$$

Dynamics

p_m: mass polarization
m_m: mass magnetization

$$g = g_0 + p_m$$

$$[p_m] = \frac{m}{s^2}$$

$$h = h_0 + m_m$$

$$[m_m] = \frac{m}{s^2}$$

Note: The definitions of p_m and m_m, compared to g and h, differ from the electrodynamic analogue concerning constants and units.

Field Equations for Polarizable/Magnetizable Matter

Electromagnetism

D: electric displacement
H: (pure) magnetic field

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_e$$

Dynamics

g_P: mechanical displacement
h_0: (pure) gravito-magnetic field

$$\nabla \cdot \mathbf{h}_0 = 0$$

$$\nabla \times \mathbf{g}_0 + \frac{1}{c}\frac{\partial \mathbf{h}_0}{\partial t} = 0$$

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m$$

$$\nabla \times \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi \mathrm{G}}{c}\mathbf{J}_m$$

ECE Field Equations of Dynamics in Momentum Representation

$$\nabla \cdot \mathbf{S} = \frac{1}{2} c V \rho_{hm} = 0 \qquad \text{(Equivalent of Gauss Law)}$$

$$\nabla \times \mathbf{L} + \frac{1}{c}\frac{\partial \mathbf{S}}{\partial t} = \frac{1}{2} V \mathbf{j}_m = 0 \qquad \text{Gravito-magnetic Law}$$

$$\nabla \cdot \mathbf{L} = \frac{1}{2} c V \rho_m = \frac{1}{2} mc \qquad \text{Newton's Law (Poisson equation)}$$

$$\nabla \times \mathbf{S} - \frac{1}{c}\frac{\partial \mathbf{L}}{\partial t} = \frac{1}{2} V \mathbf{J}_m = \frac{1}{2}\mathbf{p} \qquad \text{(Equivalent of Ampère-Maxwell Law)}$$

None of these Laws is known in the standard model.

Physical Units

Fields and Currents

L: orbital angular momentum **S**: spin angular momentum

p: linear momentum

ρ_m: mass density

J_m: mass current

V: volume of space [m^3]

ρ_{mh}: gravito-magn. mass density

j_{mh}: gravito-magn. mass current

m: mass=integral of mass density

Fields

$$[\mathbf{L}] = [\mathbf{S}] = \frac{kg \cdot m^2}{s}$$

$$[\mathbf{p}] = \frac{kg \cdot m}{s}$$

Mass Density/Current

$$[\rho_m] = \frac{kg}{m^3}$$

$$[J_m] = \frac{kg}{m^2 s}$$

„Gravito-magnetic“ Density/Current

$$[\rho_{mh}] = \frac{kg}{m^3}$$

$$[j_m] = \frac{kg}{m^2 s}$$