DEFINITIVE PROOF 1: ANTISYMMETRY OF CONNECTION.

In this definitive proof the antisymmetry of the connection is derived, and the consequences for the standard model are shown. By definition, the commutator of covariant derivatives is antisymmetric in its indices.

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] = D_\mu D_\nu - D_\nu D_\mu \tag{1}$$

Thus:

$$[D_\mu, D_\nu] = \hat{O} \qquad \text{if} \quad \mu = \nu \tag{2}$$

Its action on the vector V^ρ in any dimension and in any spacetime is:

$$[D_\mu, D_\nu]\; V^\rho = R^\rho_{\sigma\mu\nu}\; V^\sigma - T^\lambda_{\mu\nu}\, D_\lambda\, V^\rho \tag{3}$$

where the torsion tensor is:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{4}$$

From the antisymmetry of the commutator (eq. (1)):

$$T^\lambda_{\mu\nu} = -T^\lambda_{\nu\mu} \tag{5}$$

It follows that the connection cannot be symmetric. In general it could be asymmetric, that means it contains symmetric and antisymmetric parts. However we will prove that no symmetric parts are possible. Using eq.(4) we can directly insert the connection into eq.(3):

$$[D_\mu, D_\nu]\; V^\rho = -\Gamma^\lambda_{[\mu\nu]} D_\lambda\, V^\rho + \ldots \tag{6}$$

At the left hand side we have a commutator which is totally antisymmetric, therefore this property also holds for the commutator of $\Gamma^\lambda_{\mu\nu}$. The symmetry of both sides must be the same. Total antisymmetry means:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \tag{7}$$

The connection from eq.(3) is antisymmetric, Q.E.D.

Now the consequences for the standard model are derived. The fundamental and catastrophic error in the standard model of the twentieth century was to assert that the torsion is zero. From eq.(4) this means that the connection was incorrectly guessed to be symmetric for ease of calculation. From eq.(2) this means in the case μ=ν:

$$\hat{O}\, V^{\rho} = O\, V^{\sigma} - O\, D_{\lambda}\, V^{\rho} \tag{8}$$

with a null operator $\hat{O}$ where:

$$V^{\rho} \neq 0 \quad , \quad D_{\lambda}\, V^{\rho} \neq 0 \tag{9}$$

If the torsion is assumed zero for arbitrary indices, it follows from symmetry conditions above that the commutator is also zero.

$$\hat{O}\, V^{\rho} = R^{\rho}_{\sigma\mu\rho}\, V^{\sigma} - O D_{\lambda} V^{\rho} \tag{10}$$

Consequently the curvature must vanish also, there is no gravitational field, reductio ad absurdum. The result is that a symmetric connection is incompatible with the commutator, therefore it is not possible to "neglect" torsion, and the standard model is inconsistent, Q.E.D.

Some comments on Proof 1.

In the following some further detailed explanations are given to clarify the above proofs. The following diagrams emphasize the placement of the antisymmetric indices. In the first diagram the antisymmetry of the connection is determined by the first term on the right hand side. In the second diagram the incorrect standard model equation is given. It is seen that in the standard model only the antisymmetry of the Riemann tensor in its last two indices can be determined, and there is nothing to determine the correct antisymmetry of the connection.

$$[D_{\mu}, D_{\nu}]\, V^{\rho} = (\, D_{\mu} D_{\nu} - D_{\nu} D_{\mu}\,)\, .V^{\rho}$$

$$= -\,(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu})\, D_{\lambda}\, V^{\rho} + (\, \partial_{\mu}\, \Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\, \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma})\, .V^{\sigma} \tag{I}$$

$$[D_{\mu}, D_{\nu}]\, V^{\rho} = ?\ (\, \partial_{\mu}\, \Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\, \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma})\, .V^{\sigma} \tag{II}$$

For each connection appearing in the curvature tensor:

$$[D_\mu, D_\nu]\ V^\rho \ = \ -\ T^\lambda_{\mu\nu} D_\lambda\ V^\rho + \ldots\ ;\ \Gamma^\lambda_{\mu\nu} \ = \ -\ \Gamma^\lambda_{\nu\mu}$$

$$[D_\mu, D_\sigma]\ V^\rho \ = \ -\ T^\lambda_{\mu\sigma} D_\lambda\ V^\rho + \ldots\ ;\ \Gamma^\lambda_{\mu\sigma} \ = \ -\ \Gamma^\lambda_{\sigma\mu}$$

$$[D_\nu, D_\sigma]\ V^\rho \ = \ -\ T^\lambda_{\nu\sigma} D_\lambda\ V^\rho + \ldots\ ;\ \Gamma^\lambda_{\nu\sigma} \ = \ -\ \Gamma^\lambda_{\sigma\nu}$$

$$[D_\mu,\ D_\lambda]\ V^\rho \ = \ -\ T^\kappa_{\mu\lambda} D_\kappa\ V^\rho + \ldots\ ;\ \Gamma^\kappa_{\mu\lambda} \ = \ -\ \Gamma^\kappa_{\lambda\mu}$$

All connections are antisymmetric. If any is symmetric, the commutator vanishes, and both the torsion and curvature are zero. Similarly:

$$[D_\rho\,, D_\sigma]\ X^{\mu_1 \ldots \mu_\kappa}_{\nu_1 \ldots \nu_l} \ = \ -\ T^\lambda_{\rho\sigma} D_\lambda\ X^{\mu_1 \ldots \mu_\kappa}_{\nu_1 \ldots \nu_l} + \ldots$$

where X is the tensor of any rank.

Some more details of Proof 1.

The key equation of the proof 1 starts with:

$$[D_\mu, D_\nu]\ \ V^\rho = \ R^\rho_{\sigma\mu\nu}\ V^\sigma \ -\ T^\lambda_{\mu\nu}\ D_\lambda\ V^\rho \tag{1}$$

where:

$$T^\lambda_{\mu\nu} \ = \ \ \Gamma^\lambda_{\mu\nu} \ -\ \ \Gamma^\lambda_{\nu\mu} \tag{2}$$

There is a one to one correspondence between the antisymmetry of the commutator and that of the torsion:

$$[D_\mu, D_\nu]\ \ V^\rho = \ -\ T^\lambda_{\mu\nu}\ D_\lambda\ V^\rho + R^\rho_{\sigma\mu\nu}\ V^\sigma \tag{3}$$

If:

$$\mu = \nu \tag{4}$$

Then:

$$[D_\mu, D_\nu]\ \ = \hat{O} \tag{5}$$

For example:

$$[D_1, D_1]\ = \ D_1\ D_1\ -\ D_1\ D_1 = \ \hat{O} \tag{6}$$

and

$$T^\lambda_{11} \ = \ 0 \tag{7}$$

So:

$$[D_1, D_1\,]\ V^\rho = -\ T^{\lambda}_{11} D_\lambda\ V^\rho\ +\ (\,\partial_1\,\Gamma^{\rho}_{1\sigma} - \partial_1\,\Gamma^{\rho}_{1\sigma} + \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma} - \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma})\ .V^\sigma$$

$$\downarrow$$

$$0\quad V^\rho = -\quad 0\ D_\lambda\ V^\rho\ +\ (\,\partial_1\,\Gamma^{\rho}_{1\sigma} - \partial_1\,\Gamma^{\rho}_{1\sigma} + \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma} - \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma})\ .V^\sigma$$

i.e. $$0 = \quad 0\quad +\ (\,\partial_1\,\Gamma^{\rho}_{1\sigma} - \partial_1\,\Gamma^{\rho}_{1\sigma} + \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma} - \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma})\ .V^\sigma$$

So: $$(\,\partial_1\,\Gamma^{\rho}_{1\sigma} - \partial_1\,\Gamma^{\rho}_{1\sigma} + \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma} - \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma})\ .V^\sigma = 0 \tag{8}$$

However: $$V^\sigma \neq 0 \tag{9}$$

So: $$(\,\partial_1\,\Gamma^{\rho}_{1\sigma} - \partial_1\,\Gamma^{\rho}_{1\sigma} + \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma} - \Gamma^{\rho}_{1\lambda}\Gamma^{\lambda}_{1\sigma}) = 0 \tag{10}$$

The Riemann tensor is zero, irrespective of any symmetry of connection. We know from eq.(11) that for μ=ν the symmetry of the connection must be symmetric, but the complete curvature tensor is **zero.**

Conclusion: For a symmetric connection the curvature tensor and the torsion tensor are zero.

Comment on antisymmetry of operators

Assume A and B being two operators. Then the commutator of both is

[A, B] = AB - BA

In case of A=B we have

[A, A] = AA - AA = 0

There is no such thing as a "symmetric part" in an operator. Therefore, antisymmetry means exclusive antisymmetry.

Flowchart 1

Standard Model

$[D_\mu, D_\nu]\, V^\rho = ?\, R^\rho_{\sigma\mu\nu}\, V^\sigma$

$\downarrow$

$R^\rho_{\sigma\mu\nu} = 0$ if $\mu = \nu$

$\downarrow$

Symmetry of $\Gamma^\rho_{\nu\sigma}$
etc. not defined

$\downarrow$

$\Gamma^\rho_{\nu\sigma} = \Gamma^\rho_{\sigma\nu}$
assumed arbitrarily

$\downarrow$

Violation of
$D_\mu\, T^{\kappa\mu\nu} = R^{\kappa\mu\nu}_\mu$

Flowchart 2

Connection

$[D_\mu, D_\nu]\, V^\rho$ = - $T^\kappa_{\mu\nu}\, D_\lambda\, V^\rho$ + $R^\rho_{\sigma\mu\nu}\, V^\sigma$

↓

$T^\kappa_{\mu\nu}$ = $\Gamma^\kappa_{\mu\nu}$ - $\Gamma^\kappa_{\nu\mu}$ → $\Gamma^\kappa_{[\mu,\nu]}$

↓

$T^\kappa_{\mu\nu}$ = - $T^\kappa_{\nu\mu}$; $\Gamma^\kappa_{\mu\nu}$ = - $\Gamma^\kappa_{\nu\mu}$

↓

If $\Gamma^\kappa_{\mu\nu}$ = $\Gamma^\kappa_{\nu\mu}$,

$[D_\mu, D_\nu]$ = 0.

↓

Symmetric connection
⇒ zero torsion <u>and</u>
zero curvature

↓

One-to-one correspondence
between connection
and commutator

↓

$[D_\mu, D_\nu]$ ⇔ $\Gamma^\kappa_{[\mu,\nu]}$

Flowchart 3

Commutator and symmetry