195 (4): Summary of Results for Cotton Metric.

The infinitesimal line element is:

$$ds^2 = AC^{1/2} c^2 dt^2 - BC^{1/2} dr^2 - C(r) d\theta^2 \quad -(1)$$

$$= c^2 d\tau^2$$

in the plane $$dz^2 = 0. \quad -(2)$$

The total energy is:

$$E = mc^2 AC^{1/2}(r) \frac{dt}{d\tau}. \quad -(3)$$

The total angular momentum is:

$$L = m C(r) \frac{d\theta}{d\tau}. \quad -(4)$$

Here $$\frac{dt}{d\tau} = \left(AC^{1/2}(r) - \frac{v^2}{c^2} \right)^{-1/2} \quad -(5)$$

where: $$v^2 = BC^{1/2}(r) \left(\frac{dr}{dt} \right)^2 + C(r) \left(\frac{d\theta}{dt} \right)^2. \quad -(6)$$

The angular velocity is:

$$\omega = \left(\frac{Lc^2}{E} \right) \frac{A}{C^{1/2}(r)} \quad -(7)$$

The orbital equation is:

$$\left(\frac{dr}{d\theta} \right)^2 = \frac{m C(r)}{BL^2} \left[\frac{1}{A} \frac{E^2}{mc^2} - C^{1/2}(r) \left(mc^2 + \frac{1}{C(r)} \frac{L^2}{m} \right) \right] \quad -(8)$$

In eq. (6):

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega \frac{dr}{d\theta} \quad -(9)$$

Therefore:

$$v^2 = BC^{1/2}(r)\left(\frac{dr}{d\theta}\right)^2\left(\frac{d\theta}{dt}\right)^2 + C(r)\left(\frac{d\theta}{dt}\right)^2$$

$$= BC^{1/2}(r)\omega^2\left(C(r) + \left(\frac{dr}{d\theta}\right)^2\right) \quad -(10)$$

So the ratio of orbital linear velocity v to orbital angular velocity ω is:

$$\left(\frac{v}{\omega}\right)^2 = BC^{3/2}(r)\left[1 + \frac{m}{BL^2}\left(\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right)\right] \quad -(11)$$

The orbital linear velocity is:

$$v^2 = \left(\frac{Lc^2}{E}\right)^2 BA^2 C^{1/2}(r)\left[1 + \frac{m}{BL^2}\left(\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right)\right] \quad -(12)$$

The gravitational time delay is given by eq. (9).

There are five unknown: A, B, C, L and E. At least five measurements are needed, and this is possible by measuring: 1) orbital angular velocity; 2) orbital linear velocity; 3) the change of r with θ in the orbital equation; 4) the change of r with t in eq. (9); 5) the ratio of time t to proper time τ in