195(6): Crothers Metric and Precessing Ellipse

The precessing ellipse is described by:

$$\frac{d\theta}{dr} = \frac{\alpha}{xr\left(\epsilon^2 r^2 - (\alpha - r)^2\right)^{1/2}} \quad -(1)$$

From the Crothers metric:

$$\frac{d\theta}{dr} = \left(\frac{m\,C(r)}{BL^2}\right)^{1/2}\left(\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right)^{-1/2} \quad -(2)$$

so:

$$\left(\frac{xr}{\alpha}\right)^2\left(\epsilon^2 r^2 - (\alpha - r)^2\right) = \frac{BL^2}{m\,C(r)}\left(\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right) \quad -(3)$$

and this is the correct way of describing the precession of the perihelion in relativity. It is one of the experiments that can be used to find A, B, $C(r)$, L and E but the element based general relativity is no longer predictive.