195(3): Calculation of Velocity and Angular Velocity

In note 195(2) the orbital equation for the Crothers metric was calculated giving the result:

$$\frac{dr}{d\theta} = \frac{m^{1/2}}{L}\left(\frac{C(r)}{B}\right)^{1/2}\left(\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right)^{1/2} \quad -(1)$$

The velocity is calculated from:

$$v = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{d\tau}\frac{d\tau}{dt} \quad -(2)$$

with

$$\frac{d\theta}{d\tau} = \frac{L}{mC(r)}, \quad \frac{d\tau}{dt} = \frac{mc^2}{E} A C^{1/2}(r) \quad -(3)$$

so

$$v = \frac{Lc^2 A}{EC^{1/2}(r)}\frac{dr}{d\theta} \quad -(4)$$

i.e.

$$\frac{dr}{dt} = \frac{Lc^2 A}{EC^{1/2}(r)}\frac{dr}{d\theta} \quad -(5)$$

which implies that the angular velocity is:

$$\boxed{\omega = \frac{d\theta}{dt} = \frac{d\theta}{dr}\frac{dr}{dt} = \frac{Lc^2 A}{EC^{1/2}(r)}.} \quad -(6)$$

where

$$ds^2 = AC^{1/2}c^2 dt^2 - BC^{1/2}dr^2 - C\, d\theta^2 \quad -(7)$$

Units Check

$C(r) = m^2, \; B = m^{-1}, \; A = m^{-1}$

$E = mc^2 A C^{1/2}(r)\, dt/d\tau$ = joules ✓

$L = m\, C(r)\, d\theta/dt$ = kgm m^2 s^{-1} ✓

d)

$$\omega = \frac{AL}{E}\frac{c^2}{C^{1/2}(r)} = \frac{Js}{J}\frac{m^{-1}}{m} m^2 s^{-2} = s^{-1}$$

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{m\,C(r)}{BL^2}\left(\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right)$$

$$= \frac{kgm\,m^2 \quad m\,J}{m^{-1}\,kgm^2\,m^4\,s^{-2}} = \frac{s^2}{kgm}\,m^2 s^{-2}\,kgm = m^2$$

The angular momentum can be expressed as:

$$\boxed{\omega = \left(\frac{Lc^2}{E}\right)\frac{A}{C^{1/2}(r)}} \quad \text{— (8)}$$

and is proportional to $A / C^{1/2}$ through the constant of motion Lc^2/E.

The next step is to apply the methods of UFT 194.