143(8): Summary of Results for Crothers Metric.

The infinitesimal line element is:

$$ds^2 = AC^{1/2}c^2dt^2 - BC^{1/2}dr^2 - C(r)\,d\theta^2$$

$$= c^2 d\tau^2 \qquad -(1)$$

in the plane

$$dZ^2 = 0. \qquad -(2)$$

The total energy is:

$$E = mc^2 AC^{1/2}(r)\frac{dt}{d\tau}. \qquad -(3)$$

The total angular momentum is:

$$L = m\,C(r)\frac{d\theta}{d\tau}. \qquad -(4)$$

Here

$$\frac{dt}{d\tau} = \left(AC^{1/2}(r) - \frac{v^2}{c^2}\right)^{-1/2} \qquad -(5)$$

where:

$$v^2 = BC^{1/2}(r)\left(\frac{dr}{dt}\right)^2 + C(r)\left(\frac{d\theta}{dt}\right)^2. \qquad -(6)$$

The angular velocity is:

$$\omega = \left(\frac{Lc^2}{E}\right)\frac{A}{C^{1/2}(r)} \qquad -(7)$$

The orbital equation is:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{m\,C(r)}{BL^2}\left[\frac{1}{A}\frac{E^2}{mc^2} - C^{1/2}(r)\Big/\left(mc^2 + \frac{1}{C(r)}\frac{L^2}{m}\right)\right]$$

$$-(8)$$

In eq. (6):

2)
$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega\frac{dr}{d\theta} \quad -(9)$$

Therefore:

$$v^2 = BC^{1/2}(r)\,\omega^2\left(\frac{dr}{d\theta}\right)^2 + \omega^2 C(r) \quad -(10)$$

$$= \omega^2\left(BC^{1/2}(r)\left(\frac{dr}{d\theta}\right)^2 + C(r)\right) \quad -(11)$$

Therefore:
$$\left(\frac{v}{\omega}\right)^2 = BC^{1/2}(r)\left(\frac{dr}{d\theta}\right)^2 + C(r) \quad -(12)$$

The orbital linear velocity may therefore be calculated from eqns. (8) and (12). The gravitational time delay is given by eq. (9).

There are five unknowns: A, B, C, L and E. At least five measurements are needed, for example: 1) orbital angular velocity; 2) orbital linear velocity; 3) the change of r wrt θ in the orbital equation; 4) the change of r wrt t in eq. (9); 5) the ratio of time t to proper time τ in eq. (5)