

224(2): Introduction of the Higgs Mechanism

This note uses the usual standard model notation.
For the scalar field the Higgs Lagrangian starts with:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*) \quad (1)$$

where
$$V = m^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \quad (2)$$

So there are two parameters, m^2 and λ right at the outset. It is assumed that:

$$m^2 < 0, \quad (3)$$

even though m has the units of mass. This is the first random or arbitrary assumption. Its expectation value is:

$$|\langle 0 | \phi | 0 \rangle|^2 = a^2 \quad (4)$$

$$\phi = \phi_1 + i\phi_2 \quad (5)$$

so by now there are five parameters:

$$m^2, \lambda, a, \phi_1 \text{ and } \phi_2.$$

Two more arbitrary assumptions are introduced, that the minima of V are "degenerate vacua" equivalent by rotation, and that physical fields are perturbation about:

$$|\phi| = a \quad (6)$$

These assumptions are taken from condensed matter theory. Then:

$$\phi(x) = \rho(x) e^{i\theta} \quad (7)$$

introducing what are randomly asserted to be

2) scalar fields, ρ and θ . So now we have:
 $m^2, \lambda, a^2, \phi_1, \phi_2, \rho$ and θ
 i.e. seven parameters. $\frac{1}{\lambda}$ found that:

$$a^2 = -\frac{m^2}{2\lambda} \quad (8)$$

Then:
$$\phi(x) = (\rho'(x) + a) e^{i\theta(x)} \quad (9)$$

where ρ' and a are arbitrarily chosen to have
~~randomly~~ vanishing vacuum expectation values.

It is already clear that all this is just
playing with mathematics, and not physics at all.

The ~~theory~~ arrives at:

$$\begin{aligned} V = & m^2 \rho'^2 + 2m^2 a \rho' + m^2 a^2 \\ & + \lambda (\rho'^4 + 4a \rho'^3 + 6a^2 \rho'^2 + 4a^3 \rho' + a^4) \\ = & \lambda ((\phi^* \phi - a^2)^2 - a^4) \end{aligned} \quad (10)$$

and

$$\begin{aligned} (\partial_\mu \phi)(\partial^\mu \phi) = & (\partial_\mu \rho')(\partial^\mu \rho') \\ & + (\rho' + a)^2 (\partial_\mu \theta)(\partial^\mu \theta) \end{aligned} \quad (11)$$

It is now asserted that the following
 quantity is a mass:

$$m_{\rho'}^2 = 4\lambda a^2 \quad (12)$$

3) There is no term in θ^2 so this field is asserted to be massless. One may as well assert that any line that does not appear in an equation is "massless".

By definition:

$$\phi = (p' + a) e^{i\theta} \quad - (13)$$

$$\text{Real } \phi = (p' + a) \cos \theta \quad - (14)$$

and p' and a are asserted to be massive. This assertion is made because they multiply a quadratic term in eq. (11). The θ particle is known as the Goldstone boson, predicted about 1960.

The theory is described as "globally invariant" under a $U(1)$ transformation:

$$\phi = e^{i\Delta} \phi \quad - (15)$$

introducing an eighth parameter Δ .

For an $SO(3)$ gauge theory:

$$|\phi_0| = (\phi_1^2 + \phi_2^2 + \phi_3^2)^{1/2} \quad - (16)$$

$$= - \left(\frac{m^2}{4\lambda} \right)^{1/2} = a, \quad - (17)$$

$$\phi_3 = \chi + a.$$

and the potential is:

$$4) V = \frac{m^2}{2} (\phi_1^2 + \phi_2^2 + (\chi + a)^2) + \lambda (\phi_1^2 + \phi_2^2 + (\chi + a)^2)^2 \quad - (18)$$

$$= 4a^2 \lambda \chi^2 + 4a \lambda \chi (\phi_1^2 + \phi_2^2 + \chi^2) + \lambda (\phi_1^2 + \phi_2^2 + \chi^2)^2 - \lambda a^4$$

It is arbitrarily asserted that the following is a "mass": $m_\chi^2 = 8a^2 \lambda \quad - (19)$

because it multiplies χ^2 . Just as arbitrarily:

$$m_{\phi_1} = m_{\phi_2} = 0 \quad - (20)$$

because there are no terms in ϕ_1^2 and ϕ_2^2 .

So for this arbitrary "playing with mathematics" there are two "golden rules":

$$m_{\phi_1} = m_{\phi_2} = 0 \quad - (21)$$

and the "scalar field" with mass $8a^2 \lambda$.

So it is the Higgs mechanism, mass is defined by random unobservables: degenerate vacua.

This gives no understanding of the nature of mass.

All this is extended to interaction with electromagnetic field of the standard model,

5) which despite many years of definitive reputation, is still A^μ . So:

$$\mathcal{L} = (\partial_\mu + ie A_\mu) \phi (\partial^\mu - ie A^\mu) \phi^* - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (22)$$

where $\phi = a + \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2)$.

So here we have: $-m^2, \lambda, a, \phi_1, \phi_2, e, A^\mu, F_{\mu\nu}$ and \mathcal{L}

i.e. nine parameters.

In this theory it is asserted that the photon becomes massive because there is a term in $A_\mu A^\mu$. So the standard model contradicts itself because it has a massive photon in this type of theory, but a massless photon in other cases. The Lagrangian (22) is:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e^2 a^2 A_\mu A^\mu + \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - 2\lambda a^2 \phi_1^2 + \sqrt{2} e a A^\mu \partial_\mu \phi_2 + \text{cubics} + \text{quartics} \quad (23)$$

So it is asserted explicitly that the field ϕ_1 acquires mass $2\lambda a^2$, and that A^μ acquires mass $e^2 a^2$. The field ϕ_2 does not acquire mass.

6) However, neither e nor a can be predicted.
 The inconvenient mixed term in " $A^\mu \partial_\mu \phi$ " is
 "removed" by a process called "gauging".
 We compare the Lagrangian (23) with that
 of the Proca equation:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_0^2 A_\mu A^\mu \quad (24)$$

where m_0 is the photon mass, so:

$$\partial_\mu F^{\mu\nu} + m_0^2 A^\nu = 0 \quad (25)$$

(Proca, 1934). So is order for the Higgs mechanism
 to be valid: $m_0^2 = e^2 a^2$. (26)

However, this equation has never been proven in
experiments.

The Lagrangian (23) comes from the
 Klein-Gordon equation for a spin zero particle
 interacting with the electromagnetic field. It is
 well known that this particle is a fermion (an
 electron), and not a boson. So Dirac was a
severe critic of the standard model.