Numerical solutions of resonance equations with non-constant restoring force

The classical resonance equation without damping has the form

(1) $$\frac{d^2\phi}{dx^2} + \kappa_0^2 \phi = f(x)$$

where Φ is a physical quantity, x a space coordinate, and f(x) is the driving force. κ_0 represents the Hooke term, which is a linear force term providing the restoration of Φ to its original position. In classical theory of forced oscillations f(x) is a periodic function of the form

(2) $$f(x) = f_0 \cos(\kappa x)$$

with wave number κ, and κ_0 is the resonance frequency, i. e. Φ tends to infinity if κ approaches κ_0. This simplified model is not sufficient for the application cases occurring in space time resonances of ECE theory. First we have a driving force which does not have the simple cosine form of (2) but can be composed by a Fourier series. For example in paper 63 it takes the form

(3) $$f(x) = f_0 \left(A \cos(2\kappa x) + B \cos(4\kappa x) + C \cos(6\kappa x) \right)$$

with a constant amplitude f_0 and coefficients A, B, C.

Secondly the restoring force has not always a constant coefficient κ_0. Even for the simplest models of spin connection resonance of magnetism it was shown in paper 65 that κ_0 is a function of space coordinates. From a mathematical standpoint it is not clear how resonance occurs in these cases, and if it is present at all. Therefore we have made a numerical model which solves the differential equation

(4) $$\frac{d^2\phi}{dx^2} + (\kappa_0(x))^2 \phi = f(x)$$

with driving forces given by Eq. (2) and (3) and functions of κ_0 defined as

(5)
$$\kappa_0 = 1 \qquad \kappa_0 = \cos\left(\tfrac{1}{2} \cdot x\right)$$
$$\kappa_0 = \cos(1 \cdot x) \qquad \kappa_0 = (\cos(1 \cdot x))^2$$
$$\kappa_0 = \cos(2 \cdot x)$$

In case of constant κ_0 (Fig. 1) we get the resonance curves of the ECE Coulomb law (see paper 63). Using an oscillating form of κ_0 gives the curves of Figs. 2-5. There are always more resonances present than in the case of a constant κ_0, even in the case of the simple cosine driving force. This means that a rich structure of resonances is to be expected from spin connection resonances in magnetostatics. This statement can ~~even~~ be extended to the electrodynamic and mechanical sector of ECE theory.

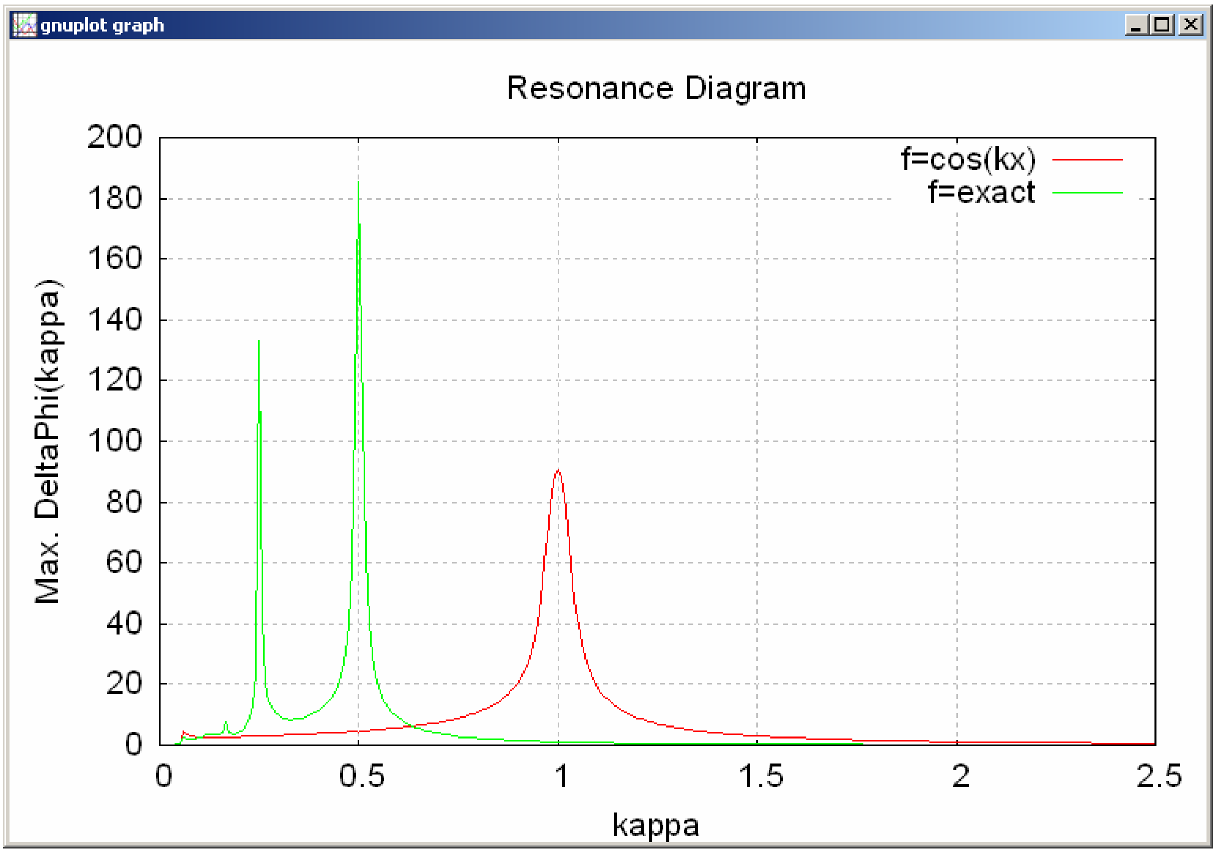

Fig. 1: $\kappa_0 = 1.$

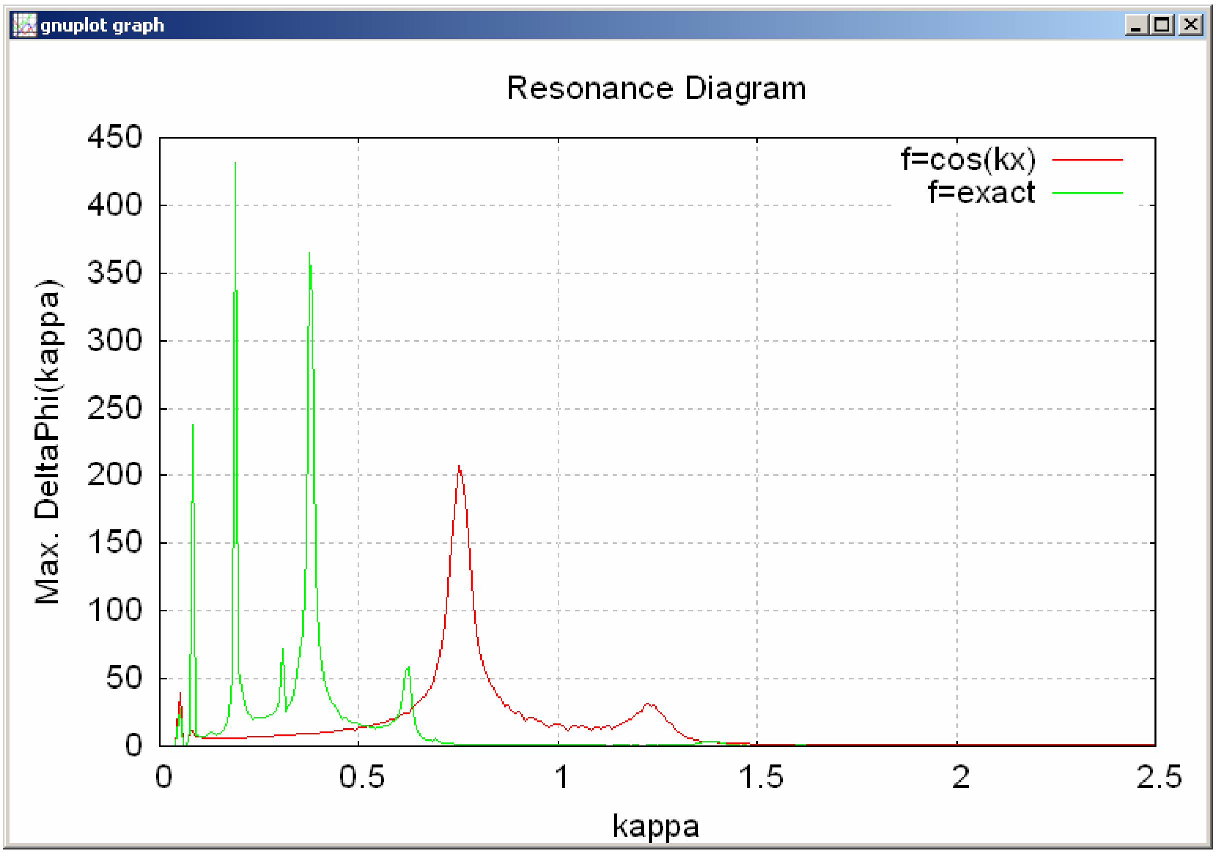

Fig. 2: κ_0 = cos (1.*r)

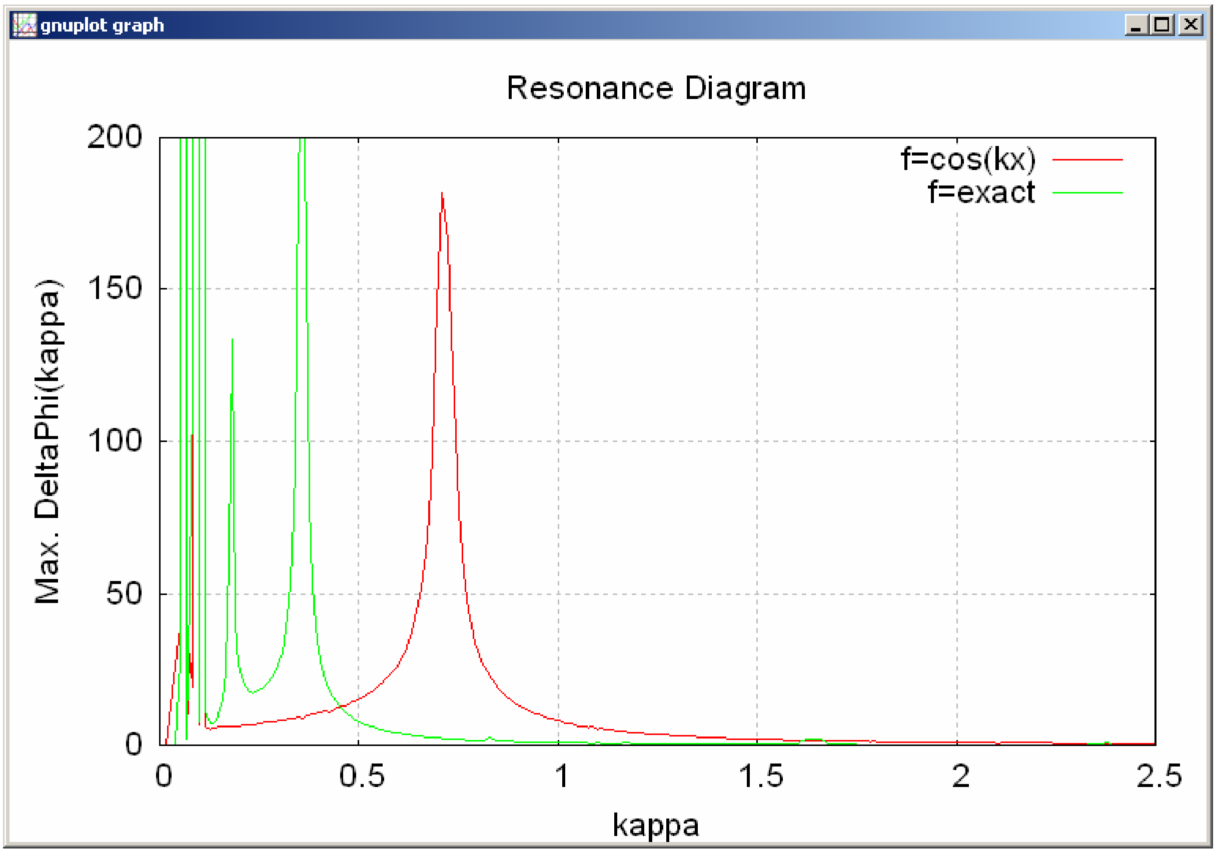

Fig. 3: κ_0 = cos (2.*r)

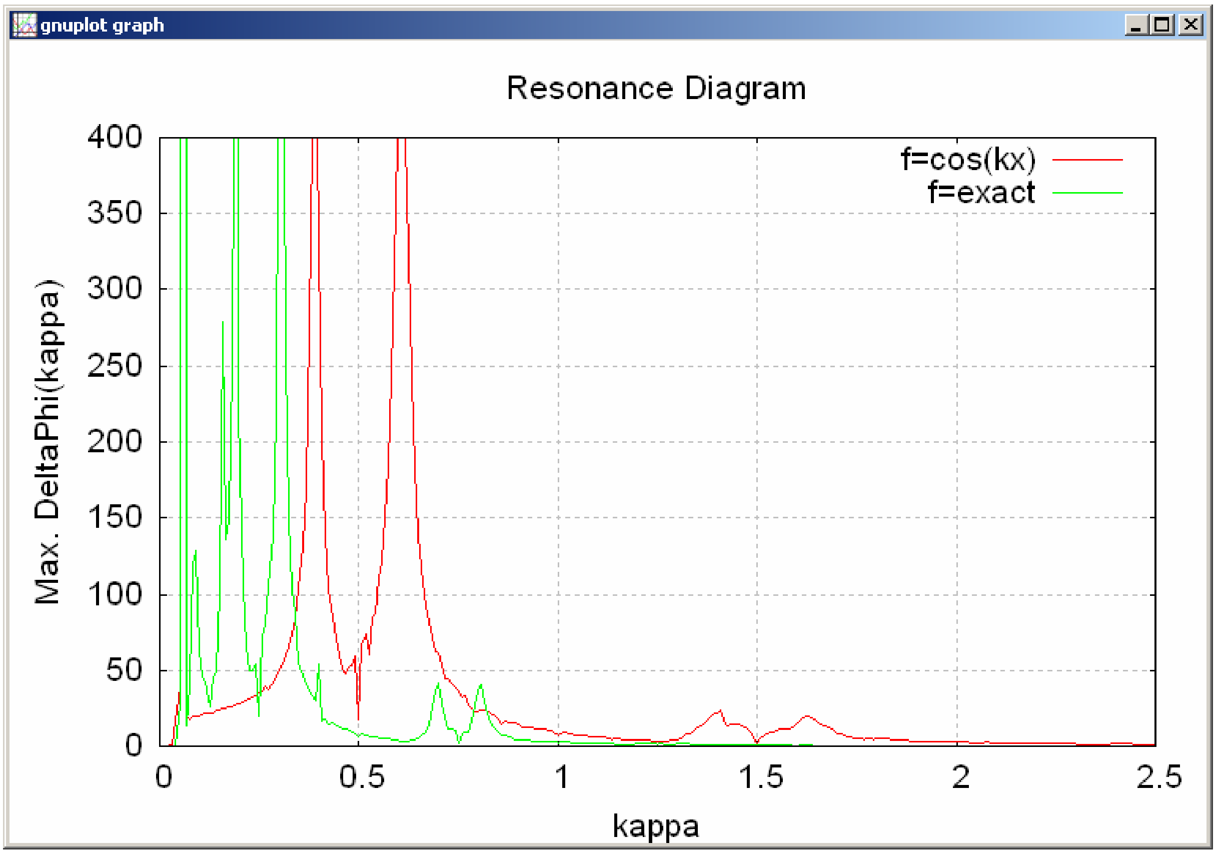

Fig. 4: κ_0 = cos (0.5*r)

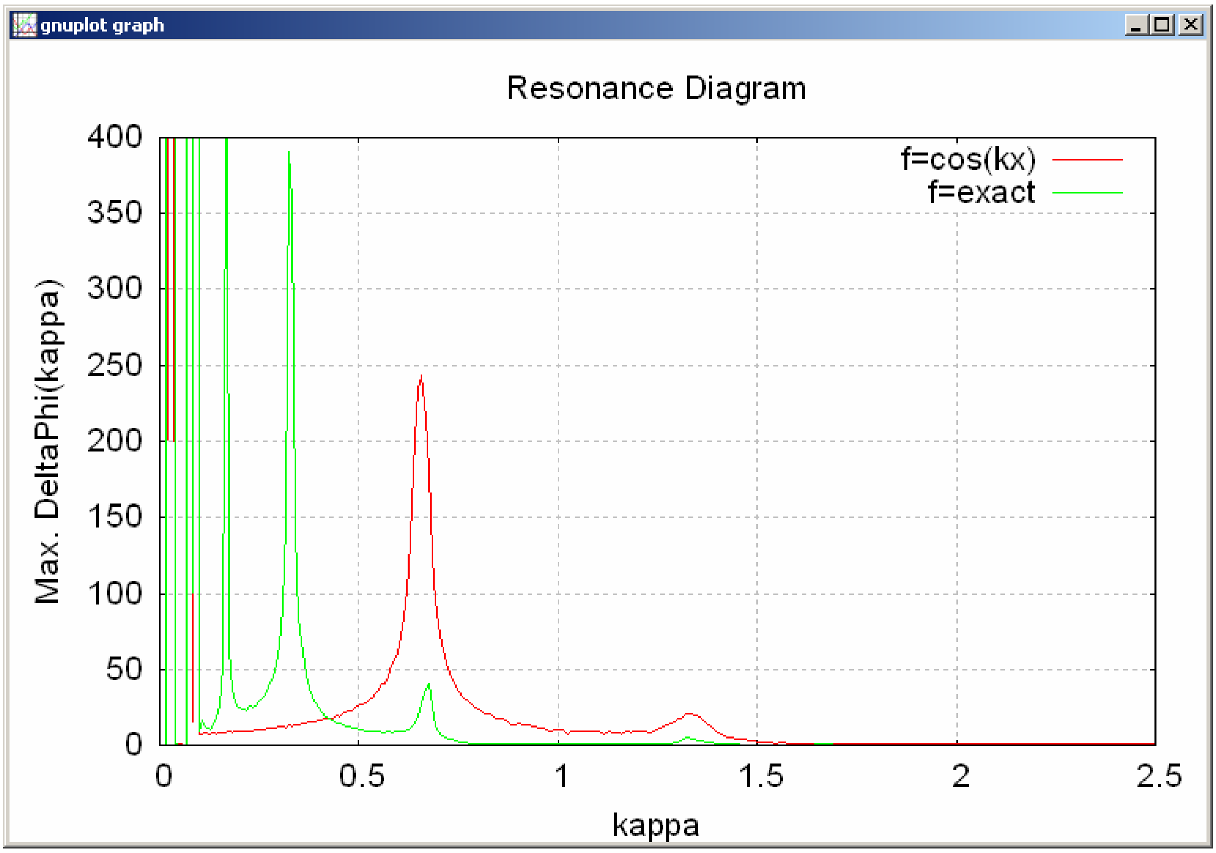

Fig. 5: κ_0 = (cos (1.*r))**2