CHAPTER TWO : ELECTRODYNAMICS AND GRAVITATION

ELECTROMAGNETIC UNITS IN S. I.

| | | |
|---|---|---|
| Electric field strength $\underline{E}$ | = | $V m^{-1} = J C^{-1} m^{-1}$ |
| Vector Potential $\underline{A}$ | = | $J s C^{-1} m^{-1}$ |
| Scalar Potential $\phi$ | = | $J C^{-1}$ |
| Vacuum permittivity $\epsilon_0$ | = | $J^{-1} C^{2} m^{-1}$ |
| Magnetic Flux Density $\underline{B}$ | = | $J s C^{-1} m^{-2}$ |
| Electric charge density $\rho$ | = | $C m^{-3}$ |
| Electric current density $\underline{J}$ | = | $C m^{-2} s^{-1}$ |
| Spacetime Torsion T | = | $m^{-1}$ |
| Spin connection $\omega$ | = | $m^{-1}$ |
| Spacetime Curvature R | = | $m^{-2}$ |

## 2.1 INTRODUCTION

The old physics, prior to the post Einsteinian paradigm shift, completely failed to provide a unified logic for electrodynamics and gravitation because the former was developed in flat or Minkowski, spacetime and the latter in a spacetime which was thought quite wrongly to be described only by curvature. The ECE theory develops both electrodynamics and gravitation directly from Cartan geometry. As shown in the ECE Engineering Model, the field equations of electrodynamics and gravitation in ECE theory have the same format, based directly and with simplicity on the underlying geometry. Therefore the Cartan geometry of chapter one is translated directly into electromagnetism and gravitation using the same type of simple, fundamental hypothesis in each case: the tetrad becomes the four potential energy and the torsion becomes the field of force.

In retrospect the method used by Einstein to translate from geometry to gravitation was cumbersome as well as being incorrect. The second Bianchi identity was reformulated by Einstein using the Ricci tensor and Ricci scalar into a format where it could be made directly proportional to the covariant Noether Theorem through the Einstein constant k. Both sides of this equation used a covariant derivative, but it was assumed by Einstein without proof that the integration constants were the same on both sides, giving the Einstein field equation:

$$G_{\mu\nu} = k T_{\mu\nu} \qquad -(1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the canonical energy momentum density, and k is the Einstein constant. This equation is completely incorrect because it uses a symmetric connection and throws away torsion. If attempts are made to correct this equation for torsion, as in UFT 88 and UFT 255, summarized in chapter one, it becomes hopelessly cumbersome, it could still only be used for gravitation and not for a unified field theory of gravitation and electromagnetism. Einstein himself thought that his field equation of 1915 could never be solved, which shows that he was bogged down in complexity. Schwarzschild provided a solution in December 1915 but in his letter declared "friendly war" on Einstein. The meaning of this is not entirely clear but obviously Schwarzschild was not satisfied with the equation. His solution did not contain singularities, and this original solution is on the net, together with a translation by Vankov of the letter to Einstein. This solution of an incorrect field equation is obviously meaningless. The errors were compounded by asserting (after Schwarzschild died in 1916) that the solution contains singularities, so the contemporary Schwarzschild metric is a misattribution and distortion, as well as being completely meaningless. It has been used endlessly by dogmatists to assert the existence of incorrect results such as big bang and black holes. So gravitational science was stagnant from 1915 to 2003. During the course of development of ECE theory it gradually became clear in papers such as UFT 150 that there were many other errors and obscurities in the Einstein theory, notably in the theory of light bending by gravitation, and in the theory of perihelion precession. One of the obvious contradictions in the theory of light deflection by gravitation is that it uses a massless photon that is nevertheless attracted to the sun. The resulting null geodesic method is full of obscurities as shown in UFT 150. The Einsteinian general relativity has been comprehensively refuted in reference (2) of chapter one. It was completely refuted experimentally in the late fifties by the discovery of the velocity curve of the whirlpool galaxy. At that point it should have been discarded, its apparent successes in the solar system are illusions. Instead, natural philosophy itself was abandoned and dark matter introduced. The Einsteinian theory is still unable to explain the velocity curve of the whirlpool galaxy, it still fails completely, and dark matter does not change this fact. So the Einstein theory cannot be meaningful in the solar system as the result of these experimental observations. The ECE theory has revealed the reason why the Einstein theory fails so badly - the neglect of torsion.

Electromagnetism also stagnated throughout the twentieth century and remained the Maxwell Heaviside theory of the nineteenth century. This theory was incorporated unchanged into the attempts of the old physics at unification using U(1) gauge invariance and the massless photon. The idea of the massless photon leads to multiple, well known problems and absurdities, notably the planar E(2) little group of the Poincaré group. Effectively this result means that the free electromagnetic field can have only two states of polarization. The two transverse states labelled (1) and (2). The time like state (0) and the longitudinal state (3) are eliminated in order to save the hypothesis of a massless photon. These problems and obscurities are explained in detail by a standard model textbook such as that of Ryder {13}. The unphysical Gupta Bleuler condition must be used to "eliminate" the (0) and (3) states, leading to multiple unsolved problems in canonical quantization. The use of the Beltrami theory as in UFT 257 onwards produces richly structured longitudinal components of the free electromagnetic field, refuting the U(1) dogma immediately and indicating the existence of photon mass. Beltrami was a contemporary of Heaviside, so the present standard model was effectively refuted as long ago as the late nineteenth century. As soon as the photon becomes identically non zero, however tiny in magnitude, the U(1) theory becomes untenable, because it is no longer gauge invariant {1 - 10}, and the Proca equation replaces the d'Alembert equation. The ECE theory leads to the Proca equation and finite photon mass from the tetrad postulate, using the same basic hypothesis as that which translates geometry into electromagnetism.

Although brilliantly successful in its time, there are many limitations of the Maxwell Heaviside (MH) theory of electromagnetism. In the field of non linear optics for example its limitations are revealed by the inverse Faraday effect {1 - 10} (IFE). This phenomenon is the magnetization of material matter by circularly polarized electromagnetic radiation. It was inferred theoretically {7} by Piekara and Kielich, and later by Pershan, and was first observed experimentally in the mid sixties by van der Ziel at al. in the Bloembergen group at Harvard. It occurs for example in one electron as in UFT 80 to 84 on www.aias.us. The old U(1) gauge invariant theory of electromagnetism becomes untenable immediately when dealing with the inverse Faraday effect because the latter is caused by the conjugate product of circularly polarized radiation, the cross product of the vector potential with its complex conjugate:

$$\underline{A} \times \underline{A}^{*} = \underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(2)$$

The indices (1) and (2) are used to define the complex circular basis {1 - 10}, whose unit vectors are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j}) \quad -(3)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j}) \quad -(4)$$

$$\underline{e}^{(3)} = \underline{k} \quad -(5)$$

obeying the cyclical, O(3) symmetry, relation:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\underline{e}^{(3)*} \quad -(6)$$

$$\underline{e}^{(3)} \times \underline{e}^{(1)} = i\underline{e}^{(2)*} \quad -(7)$$

$$\underline{e}^{(2)} \times \underline{e}^{(3)} = i\underline{e}^{(1)*} \quad -(8)$$

in three dimensional space. The unit vectors $\underline{e}^{(1)}$ and $\underline{e}^{(2)}$ are complex conjugates. The gauge principle of the MH theory can be expressed as follows:

$$\underline{A} \rightarrow \underline{A} + \underline{\nabla}\chi \quad -(9)$$

so the conjugate product becomes:

$$\underline{A} \times \underline{A}^{*} = (\underline{A} + \underline{\nabla}\chi) \times (\underline{A} + \underline{\nabla}\chi)^{*} \quad -(10)$$

and is not U(1) gauge invariant, so the resulting longitudinal magnetization of the inverse Faraday effect is not gauge invariant, QED. Many other phenomena in non linear optics {7} are not U(1) gauge invariant and they all refute the standard model and such artifacts as the "Higgs boson". The absurdity of the old physics becomes glaringly evident in that it asserts that the conjugate product exists in isolation of the longitudinal and time like components of spacetime, (0) and (3). So in the old physics the cross product ( $\wedge$ ) cannot produce a longitudinal component. This is absurd because space has three components (1), (2) and (3). The resolution of this fundamental paradox was discovered in Nov. 1991 with the inference of the B(3) field, the appellation given to the longitudinal magnetic component of the free electromagnetic field, defined by {1 - 10}:

$$\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(11)$$

where g is a parameter.

The B(3) field is the key to the geometrical unification of gravitation and electromagnetism and also infers the existence of photon mass experimentally, because it is longitudinal and observable experimentally in the inverse Faraday effect. The zero photon mass theory is absurd because it asserts that B(3) cannot exist, that the third component of space itself cannot exist, and that the inverse Faraday effect does not exist. The equation that defines the B(3) field is not U(1) gauge invariant because the B(3) field is changed by the gauge transform ( 10 ). The equation is not therefore one of U(1) electrodynamics, and was used in the nineties to develop a higher topology electrodynamics known as O(3) electrodynamics {1 - 10}. These papers are recorded in the Omnia Opera section of www.aias.us. Almost simultaneously, several other theories of higher topology electrodynamics were developed {7}, notably theories by Horwitz et al., Lehnert and Roy, Barrett, and Harmuth et al.,and by Evans and Crowell {8} These are described in several volumes of the "Contemporary Chemical Physics" series edited by M. W. Evans {14}. These higher topology electrodynamical theories also occur in Beltrami theories as reviewed for example by Reed {7}. In 2003 these higher topology theories evolved into ECE theory.

## 2.2 THE FUNDAMENTAL HYPOTHESES AND FIELD AND WAVE EQUATIONS.

The first hypothesis of Einstein Cartan Evans (ECE) unified field theory is that the electromagnetic potential ( $A^a_\mu$ ) is the Cartan tetrad within a scaling factor. Therefore the electromagnetic potential is defined by:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(12)$$

and has one upper index a, indicating the state of polarization, and one lower index to indicate that it is a vector valued differential one-form of Cartan's geometry. The gravitational potential is defined by:

$$\Phi^a_\mu = \Phi^{(0)} q^a_\mu \quad -(13)$$

where $\Phi^{(0)}$ is a scaling factor. Therefore the first ECE hypothesis means that electromagnetism is Cartan's geometry within a scalar $A^{(0)}$. Physics is geometry. Ubi materia, ibi geometria (Johannes Kepler). This is a much simpler hypothesis than that of Einstein, and much more powerful. It is a hypothesis that extends general relativity to electromagnetism. The mathematical correctness of the theory is guaranteed by the mathematical correctness and economy of thought of Cartan's geometry as described in chapter one.

The second ECE hypothesis is that the electromagnetic field ( $F^a_{\mu\nu}$ ) is the Cartan torsion within the same scaling factor as the potential. The second hypothesis follows from the first hypothesis by the first Cartan Maurer structure equation. Therefore in minimal notation:

$$F = D \wedge A = d \wedge A + \omega \wedge A \quad -(14)$$

which is an elegant relation between field and potential, the simplest possible relation in a geometry with both torsion and curvature. The field is the covariant wedge derivative of the potential, both for electromagnetism and gravitation. It follows that the entire geometrical development of chapter one can be applied directly to electromagnetism and gravitation. In the standard notation of differential geometry used by S. M. Carroll {11} the electromagnetic field is defined in ECE theory by:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b. \quad -(15)$$

In MH theory the same relation is {13}:

$$F = d \wedge A. \quad -(16)$$

The MH theory does not have a spin connection and does not have polarization indices. The ECE theory is general relativity based directly on Cartan geometry, the MH theory is special relativity and is not based on geometry. The presence of the spin connection in Eq. ( 15 ) means that the field is the frame of reference itself, a dynamic frame that translates and rotates. In MH theory the field is an entity different in concept from the frame of reference, the Minkowski frame of flat spacetime.

In tensor notation the electromagnetic field is:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu \quad -(17)$$

and can be expressed more simply as:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + A^{(0)} \left( \omega^a_{\mu\nu} - \omega^a_{\nu\mu} \right). \quad -(18)$$

In the MH theory the electromagnetic field is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(19)$$

and has no polarization index or spin connection. The electromagnetic potential is the four vector:

$$A^a_\mu = \left( A^a_0, -\underline{A}^a \right) = \left( \frac{\phi^a}{c}, -\underline{A}^a \right) - (20)$$

in covariant definition, or:

$$A^{a\mu} = \left( A^{a0}, \underline{A}^a \right) = \left( \frac{\phi^a}{c}, \underline{A}^a \right) - (21)$$

in contravariant definition. The upper index a denotes the state of polarization. For example in the complex circular basis it has four indices:

$$a = (0),\ (1),\ (2),\ (3) \quad -(23)$$

one timelike (0) and three spacelike (1), (2), (3). The (1) and (2) indices are transverse and the (3) index is longitudinal. The spacelike part of the potential four vector is the vector $\underline{A}^a$ and so this can only have space indices, (1), (2) and (3). It cannot have a timelike index (0) by definition. The four potential can be written for each of the four indices (0), (1), (2) and (3) as:

$$A_\mu = (A_0, -\underline{A}). \quad -(24)$$

When the a index is (0) the four potential reduces to the scalar potential:

$$A_\mu^{(0)} = (A_0^{(0)}, \underline{0}) \quad -(25)$$

when the a index is (1), (2) or (3) the four potential is interpreted as:

$$A_\mu^{(i)} = (A_0^{(i)}, -\underline{A}^{(i)}), \quad i = 1, 2, 3 \quad -(26)$$

so $A_0^{(1)}$ for example is the scalar part of the four potential $A_\mu^a$ associated with index (1). As described by S. M. Carroll, the tetrad is a one form for each index a. This means that the four potential $A_\mu^a$ is a four potential for each index a:

$$A_\mu^{(0)} = (A_\mu)^{(0)} \quad -(27)$$

$$A_\mu^{(i)} = (A_\mu)^{(i)}, \quad i = 1, 2, 3 \quad -(28)$$

and this is a basic property of Cartan geometry.

In order to translate the tensor notation of Eq. ( 17 ) to vector notation, it is necessary to define the torsion as a four by four antisymmetric matrix. The choice of matrix is guided by experiment, so that the ECE theory reduces to laws that are able to describe electromagnetic phenomena by direct use of Cartan geometry. As described in chapter one there exist orbital and spin torsion defined by equations which are similar in structure to electromagnetic laws which have been tested with great precision, notably the Gauss law of magnetism, the Faraday law of induction, the Coulomb law and the Ampère Maxwell law. These laws must be recovered in a well defined limit of ECE theory. Newtonian gravitation must be recovered in another limit of ECE theory.

The torsion matrix for each a is chosen by hypothesis to be:

$$T_{\rho\sigma} = \begin{bmatrix} 0 & T_1(\text{orb}) & T_2(\text{orb}) & T_3(\text{orb}) \\ -T_1(\text{orb}) & 0 & -T_3(\text{spin}) & T_2(\text{spin}) \\ -T_2(\text{orb}) & T_3(\text{spin}) & 0 & -T_1(\text{spin}) \\ -T_3(\text{orb}) & -T_2(\text{spin}) & T_1(\text{spin}) & 0 \end{bmatrix} \quad -(29)$$

This equation may be looked upon as the third ECE hypothesis. The Hodge dual {1 - 11, 13} of this matrix is:

$$\tilde{T}^{\mu\nu} = \begin{bmatrix} 0 & -T^1(\text{spin}) & -T^2(\text{spin}) & -T^3(\text{spin}) \\ T^1(\text{spin}) & 0 & T^3(\text{orb}) & -T^2(\text{orb}) \\ T^2(\text{spin}) & -T^3(\text{orb}) & 0 & T^1(\text{orb}) \\ T^3(\text{spin}) & T^2(\text{orb}) & -T^1(\text{orb}) & 0 \end{bmatrix} \quad -(30)$$

Indices are raised and lowered by the metric tensor in any space {11}:

$$\tilde{T}^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} \tilde{T}_{\alpha\beta} \quad -(31)$$

Alternatively the antisymmetric torsion matrix may be defined as:

$$T^{\mu\nu} = \begin{bmatrix} 0 & -T^1(orb) & -T^2(orb) & -T^3(orb) \\ T^1(orb) & 0 & -T^3(spin) & T^2(spin) \\ T^2(orb) & T^3(spin) & 0 & -T^1(spin) \\ T^3(orb) & -T^2(spin) & T^1(spin) & 0 \end{bmatrix} \quad -(32)$$

with raised indices. From this definition the spin torsion vector in three dimensions is:

$$\underset{\sim}{T}(spin) = T_X(spin)\,\underline{i} + T_Y(spin)\,\underline{j} + T_Z(spin)\,\underline{k} \quad -(33)$$

in which:

$$T_X(spin) = T^1(spin) = \tilde{T}^{10} = -\tilde{T}^{01} \quad -(34)$$

$$T_Y(spin) = T^2(spin) = \tilde{T}^{20} = -\tilde{T}^{02} \quad -(35)$$

$$T_Z(spin) = T^3(spin) = \tilde{T}^{30} = -\tilde{T}^{03} \quad -(36)$$

Similarly the orbital torsion vector in three dimensions is defined by:

$$\underset{\sim}{T}(orb) = T_X(orb)\,\underline{i} + T_Y(orb)\,\underline{j} + T_Z(orb)\,\underline{k} \quad -(37)$$

where the vector components are related to the matrix components as follows:

$$T_X(orb) = T^1(orb) = T_{10} = -T_{01} \quad -(38)$$

$$T_Y(orb) = T^2(orb) = T_{20} = -T_{02} \quad -(39)$$

$$T_Z(orb) = T^3(orb) = T_{30} = -T_{03} \quad -(40)$$

With these definitions the electric field strength $\underline{E}^a$ and the magnetic flux density $\underline{B}^a$ are

defined by:

$$\underline{E}^a = cA^{(0)}\underset{\sim}{T}^a(orb) \quad -(41)$$

and

$$\underline{B}^{a} = A^{(0)} \underset{\sim}{I}^{a}(\text{spin}). \quad -(42)$$

For each index a the field tensor with raised $\mu$ and $\nu$ is defined to be:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix}. \quad -(43)$$

With these fundamental definitions the tensor notation ( 17 ) can be translated to vector notation. The latter is used by engineers and is more transparent than tensor notation. The four derivative appearing in the tensor equation ( 17 ) is defined to be:

$$\partial_\mu = \left( \frac{1}{c}\frac{\partial}{\partial t}, \underset{\sim}{\nabla} \right). \quad -(44)$$

Consider the indices of the orbital torsion:

$$T^{a}_{0i} = \partial_0 q^{a}_{i} - \partial_i q^{a}_{0} + \omega^{a}_{0b} q^{b}_{i} - \omega^{a}_{ib} q^{b}_{0}$$
$$i = 1, 2, 3. \quad -(45)$$

These translate into the indices of the field tensor as follows:

$$F^{a}_{0i} = \partial_0 A^{a}_{i} - \partial_i A^{a}_{0} + \omega^{a}_{0b} A^{b}_{i} - \omega^{a}_{ib} A^{b}_{0}$$
$$i = 1, 2, 3 \quad -(46)$$

from which it follows that the electric field strength is:

$$\underline{E}^{a} = -c\underline{\nabla} A^{a}_{0} - \frac{\partial \underline{A}^{a}}{\partial t} - c\omega^{a}_{0b}\underline{A}^{b} + cA^{b}_{0}\underline{\omega}^{a}_{b} \quad -(47)$$

where the spin connection four vector is expressed as:

$$\omega^a_{\mu b} = \left( \omega^a_{0b} , -\underline{\omega}^a_b \right) \quad -(48)$$

using the above definitions.

The indices of the spin torsion tensor are:

$$\begin{aligned} T^a_{12} &= \partial_1 q^a_2 - \partial_2 q^a_1 + \omega^a_{1b} q^b_2 - \omega^a_{2b} q^b_1 \\ T^a_{13} &= \partial_1 q^a_3 - \partial_3 q^a_1 + \omega^a_{1b} q^b_3 - \omega^a_{3b} q^b_1 \\ T^a_{23} &= \partial_2 q^a_3 - \partial_3 q^a_2 + \omega^a_{2b} q^b_3 - \omega^a_{3b} q^b_2 \end{aligned} \quad -(49)$$

and they translate into the spin components of the field tensor:

$$\begin{aligned} F^a_{12} &= \partial_1 A^a_2 - \partial_2 A^a_1 + \omega^a_{1b} A^b_2 - \omega^a_{2b} A^b_1 \\ F^a_{13} &= \partial_1 A^a_3 - \partial_3 A^a_1 + \omega^a_{1b} A^b_3 - \omega^a_{3b} A^b_1 \\ F^a_{23} &= \partial_2 A^a_3 - \partial_3 A^a_2 + \omega^a_{2b} A^b_3 - \omega^a_{3b} A^b_3 . \end{aligned} \quad -(50)$$

With the above definitions these equations can be expressed as the magnetic flux density:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a_b \times \underline{A}^b \quad -(51)$$

In the MH theory the corresponding equations are:

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} \quad -(52)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(53)$$

without polarization indices and without spin connection.

2.3 THE B(3) FIELD IN CARTAN GEOMETRY.

The B(3) field is a consequence of the general expression for magnetic flux density in ECE theory:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b . \quad -(54)$$

In general, summation over repeated indices means that:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_{(1)} \times \underline{A}^{(1)} - \underline{\omega}^a{}_{(2)} \times \underline{A}^{(2)} - \underline{\omega}^a{}_{(3)} \times \underline{A}^{(3)} \quad -(55)$$

but this general expression can be simplified as discussed later in this book using the assumption:

$$\underline{\omega}^a{}_b = \epsilon^a{}_{bc}\, \underline{\omega}^c \quad -(56)$$

which is the expression for the duality of a tensor and vector. It can be shown using the vector form of the Cartan identity that the B(3) field is given by:

$$\underline{B}^{(3)} = \underline{\nabla} \times \underline{A}^{(3)} - \frac{i\kappa}{A^{(0)}}\, \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(57)$$

where the potentials are related by the cyclic theorem:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i A^{(0)} \underline{A}^{(3)*} . \quad -(58)$$

$$\text{et cyclicum}$$

For plane wave the potentials are as follows:

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}} \left( \underline{i} - i \underline{j} \right) e^{i(\omega t - \kappa Z)}, \quad -(59)$$

$$\underline{A}^{(3)} = A^{(0)} \underline{k} , \quad -(60)$$

so the B(3) field is defined by:

$$\underline{B}^{(3)} = -\frac{i\kappa}{A^{(0)}}\,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(61)$$

Therefore B(3) is the result of general relativity, and does not exist in the Maxwell Heaviside field theory because the MH theory is a theory of special relativity without a geometrical connection. The B(3) field is a radiated longitudinal field that propagates in the (3) or Z axis. When it was inferred in Nov. 1991 it was a completely new concept, and it was gradually realized that it led to a higher topology electrodynamics which was identified with Cartan geometry in 2003. "Higher topology" in this sense means that a different differential geometry is needed to define electrodynamics. This can be seen through the fact that the field in U(1) gauge invariant electrodynamics is:

$$F = d \wedge A \quad -(62)$$

but in ECE theory it is:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(63)$$

with the presence of indices and spin connection. A choice of internal indices leads to O(3) electrodynamics as outlined above and explained in more detail later.

It was gradually realized that O(3) electrodynamics and ECE electrodynamics accurately reduce to the MH theory in certain limits, but also give much more information, an example being the inverse Faraday effect. The B(3) field led for the first time to an electrodynamics that is based on general covariance, and not Lorentz covariance, so it became easily possible to unify electromagnetism with gravitation.

It is important to realize that B(3) is not a static magnetic field, it interacts with material matter through the conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$ by which it is defined. So B(3) is intrinsically non linear in nature while a static magnetic field is not related to the conjugate product of non linear optics. The B(3) field needs for its definition a geometrical connection, and a different set of field equations from those that govern a static magnetic field. The latter is governed by the Gauss law of magnetism and the Ampere law. The static magnetic field does not propagate at c in the vacuum, but B(3) propagates in the vacuum along with A(1). and A(2) and when B(3) interacts with matter it produces a magnetization through a well defined hyperpolarizability in the inverse Faraday effect. The field equations needed to define B(3) must be obtained from Cartan geometry, and are not equations of Minkowski spacetime.

## 2.4. THE FIELD EQUATIONS OF ELECTROMAGNETISM.

These are based directly on the Cartan and Evans identities using the hypotheses ( 41 ) and ( 42) and give a richly structured theory summarized in the ECE Engineering Model on www.aias.us. Before proceeding to a description of the field equations a summary is given of the Cartan identity in vector notation. In a similar manner to the torsion, the second Cartan Maurer structure equation gives an orbital curvature and a spin curvature:

$$\underline{R}^a{}_b(\text{spin}) = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b . \quad -(64)$$

As in UFT 254 consider now the Cartan identity:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b . \quad -(65)$$

The space part of this identity can be written as:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{q}^b \cdot \left( \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right) \quad -(66)$$

Re arranging and using:

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{\omega}^a{}_b \cdot \underline{\omega}^a{}_c \times \underline{q}^c \quad -(67)$$

and $$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a = 0 \qquad -(68)$$

gives:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \qquad -(69)$$

i.e. gives the Cartan identity in vector notation, a very useful result that will be used later in this chapter and book. The self consistency and correctness of the result ( 69) is shown by the fact that it is an example of the well known vector identity:

$$\underline{\nabla} \cdot \underline{F} \times \underline{G} = \underline{G} \cdot \underline{\nabla} \times \underline{F} - \underline{F} \cdot \underline{\nabla} \times \underline{G}. \qquad -(70)$$

So it can be seen clearly that Cartan geometry generalizes well known geometry and vector identities.

For ECE electrodynamics Eq. ( 69) becomes:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b - \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b. \qquad -(71)$$

The magnetic flux density is defined in ECE theory as:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \qquad -(72)$$

so:

$$\underline{\nabla} \cdot \underline{B}^a = -\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b \qquad -(73)$$

giving the Gauss law of magnetism in general relativity and ECE unified field theory.

As in UFT 256 the Cartan identity and the fundamental ECE hypotheses give the homogeneous field equations of electromagnetism in ECE theory:

$$\underline{\nabla} \cdot \underline{B}^a = \frac{\rho_m}{\epsilon_0 c} = \underline{\omega}^a{}_b \cdot \underline{B}^b - \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \qquad -(74)$$

and

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = \underline{J}^m / \epsilon_0$$
$$= \underline{\omega}^a{}_b \times \underline{E}^b - c\omega_0 \underline{B}^a - c\left(\underline{A}^b \times \underline{R}^a{}_b(\text{orb}) - A_0^b \underline{R}^a{}_b(\text{spin})\right) \quad -(75)$$

in which the spin curvature is defined by Eq. ( 64 ) and the orbital curvature by:

$$\underline{R}^a{}_b(\text{orb}) = -\underline{\nabla}\,\omega^a{}_{0b} - \frac{1}{c}\frac{\partial \underline{\omega}^a{}_b}{\partial t} - \omega^a_{0c}\,\underline{\omega}^c{}_b + \omega^c_{0b}\,\underline{\omega}^a{}_c . \quad -(76)$$

The right hand sides of these equations give respectively the magnetic charge density and the magnetic current density. The controversy over the existence of the magnetic charge current density has been going on for over a century, and the consensus seems to be that they do not exist. (If they are proven to be reproducible and repeatable the ECE theory can account for them as in the above equations.) If the magnetic charge current density vanishes then:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(77)$$

and

$$\underline{\omega}^a{}_b \times \underline{E}^b - c\omega_0 \underline{B}^a = c\left(\underline{A}^b \times \underline{R}^a{}_b(\text{orb}) - A_0^b \underline{R}^a{}_b(\text{spin})\right) \quad -(78)$$

and they imply the Gauss law of magnetism in ECE theory:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(79)$$

and the Faraday law of induction:

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = \underline{0} . \quad -(80)$$

The Evans identity gives

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\,\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(81)$$

and:
$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a$$
$$= \underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c} \underline{E}^b - A_0^b \underline{R}^a{}_b(\text{orb}) - \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) \quad (82)$$

Eq. ( 81 ) defines the electric charge density:

$$\rho^a = \epsilon_0 \left( \underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \right) \quad -(83)$$

and Eq. ( 82 ) defines the electric current density:

$$\underline{J}^a = \frac{1}{\mu_0} \left( \underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c} \underline{E}^b - \left( \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) + A_0^b \underline{R}^a{}_b(\text{orb}) \right) \right) \quad -(84)$$

With these definitions the inhomogeneous field equations become the Coulomb law:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(85)$$

and the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(86)$$

2.5 THE FIELD EQUATIONS OF GRAVITATION.

As shown in the Engineering Model the field equations of gravitation are the two homogeneous field equations:

$$\underline{\nabla} \cdot \underline{h} = 4\pi G \rho_m \quad -(87)$$

and

$$\underline{\nabla} \times \underline{g} + \frac{1}{c} \frac{\partial \underline{h}}{\partial t} = \frac{4\pi G}{c} \underline{j}_m \quad -(88)$$

and the two inhomogeneous equations:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(89)$$

and

$$\underline{\nabla} \times \underline{h} - \frac{1}{c}\frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c}\underline{J}_m . \quad -(90)$$

Here $\underline{g}$ is the acceleration due to gravity, and $\underline{h}$ is the gravitomagnetic field, defined by the Cartan Maurer structure equations as:

$$\underline{g} = -\frac{\partial \underline{Q}}{\partial t} - \underline{\nabla}\,\underline{\Phi} - \omega_0 \underline{Q} + \underline{\Phi}\,\underline{\omega} \quad -(91)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} . \quad -(92)$$

In the Newtonian physics only Eq. ( 89 ) exists, where G is the Newton constant and where $\rho_m$ is the mass density. In the ECE equations there is a gravitomagnetic field $\underline{h}$ (developed in UFT 117 and UFT 118) and a Faraday law of gravitational induction, Eq. ( 88 ), developed in UFT 75. The latter paper describes the experimental evidence for the gravitational law of induction and UFT 117 and UFT 118 use the gravitomagnetic field to explain precession not explicable in the Newtonian theory.

It is likely that all the fields predicted by the ECE theory of gravitation will eventually be discovered because they are based on geometry as advocated by Kepler. During the course of the development of ECE there have been many advances in electromagnetism and gravitation. There has been space here for a short overview summary.