PROOF OF THE ANTISYMMETRY OF THE CHRISTOFFEL CONNECTION USING THE BASIC THEOREMS OF DIFFERENTIAL GEOMETRY.

by

M .W. Evans, H. Eckardt and D. W. Lindstrom,

Civil List, AIAS and UPITEC,

(www.aias.us, www.webarchive.org.uk, www.atomicprecision.com, www.et3m.net, www.upitec.org)

ABSTRACT

The Christoffel connection is proven rigorously to be antisymmetric by consideration of the complete set of equations of differential geometry, combined with consideration of the general coordinate transformation. The complete set of equations available include the two structure equations, the Cartan identity and the tetrad postulate. A further rigorous proof of antisymmetry is given by consideration of the inhomogeneous term in its general coordinate transformation. It is proven that this term is zero, so the connection can have no symmetric component in any frame of reference. These proofs all refute the Einsteinian general relativity, which is incorrectly based on a symmetric connection.

Keywords: ECE theory, antisymmetry of the Christoffel connection, refutation of Einsteinian general relativity.

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1. INTRODUCTION

It has been known for almost a hundred years {1 - 10} that the Einsteinian general relativity is incorrect. It was first refuted by Schwarzschild on December 22nd 1915, a month after the publication of Einstein's first paper on the perihelion precession. A translation and analysis by Vankov {11} of the Schwarzschild paper is available on the net. In ref. (1) several clear and simple refutations of the theory are given. In Section 2, it is proven straightforwardly that the Christoffel connection must be antisymmetric. Its antisymmetry refutes the Einsteinian general relativity (EGR), whch used a symmetric connection because at the time of development of EGR (1905 - 1915) the concept of torsion was unknown. Torsion was inferred by Cartan {12} in the early twenties, and its inference effectively refuted EGR at that time. In Section 2 this is made clear by combined consideration of the basic theorems available in differential geometry, the definitions of torsion and curvature, the Cartan identity, the tetrad postulate, and the general coordinate transformation of the connection. The Einstein Cartan Evans (ECE) theory is the only one at present that correctly accounts for the connection antisymmetry.

2. PROOF OF CONNECTION ANTISYMMETRY

Assume that the Christoffel connection is in general asymmetric:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\nu\mu}\right) + \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right) \quad -(1)$$

with symmetric and antisymmetric components. Eq. (1) is a well known theorem of matrices. Now note that the Cartan identity is true for eq. (1). The Cartan identity is an exact identity consisting of a cyclical sum of terms on both sides. In UFT209 it was given in Riemannian format:

$$R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\nu\rho\mu} := \partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}{}_{\mu\sigma} T^{\sigma}{}_{\nu\rho}$$
$$+ \partial_\rho T^{\lambda}{}_{\mu\nu} + \Gamma^{\lambda}{}_{\rho\sigma} T^{\sigma}{}_{\mu\nu} \quad -(2)$$
$$+ \partial_\nu T^{\lambda}{}_{\rho\mu} + \Gamma^{\lambda}{}_{\nu\sigma} T^{\sigma}{}_{\rho\mu} .$$

The sum of curvature tensors appears on the left hand side, and the sum on the right hand side is made up of derivatives of torsion and products of connection and torsion in cyclical permutation. The left and right hand sides of eq. (2) are identically the same. The following is a mathematical solution of the identity:

$$R^{\lambda}{}_{\mu\nu\rho} = \partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}{}_{\mu\sigma} T^{\sigma}{}_{\nu\rho} . \quad -(3)$$

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Now note that the curvature and torsion may be defined by the commutator of covariant derivatives acting on a vector V^ρ :

$$[D_\mu , D_\nu] V^\rho = R^{\rho}{}_{\sigma\mu\nu} V^\sigma - T^{\lambda}{}_{\mu\nu} D_\lambda V^\rho \quad -(4)$$

where

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda} \Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda} \Gamma^{\lambda}{}_{\mu\sigma} \quad -(5)$$

and

$$T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu} . \quad -(6)$$

In general, the connection in Eqs. (4) to (6) is defined by eq. (1). This is also true for eq. (2). This set of equations is rigorously self-consistent as is well known {12}.

Therefore the curvature tensor defined by eqs. (3) and (5) is the same curvature tensor. It follows that:

$$R^{\lambda}{}_{\mu\nu\rho} = \partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}{}_{\mu\sigma} T^{\sigma}{}_{\nu\rho} \qquad -(7)$$

$$= \partial_\mu \Gamma^{\lambda}{}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\rho} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\mu\rho}$$

If the connection is symmetric the torsion vanishes form Eq. (6), and form Eq. (7) the curvature vanishes also, reductio ad absurdum. Therefore the connection is antisymmetric:

$$\Gamma^{\lambda}{}_{\mu\nu} = -\Gamma^{\lambda}{}_{\nu\mu} . \qquad -(8)$$

For non zero torsion and curvature the connection is antisymmetric, Q. E. D.

This proof is confirmed by considerations of Eq. (4). If in that equation the connection is assumed to be symmetric, then:

$$\mu = \nu \qquad -(9)$$

in which case:

$$[D_\mu, D_\nu] V^\rho = 0 \qquad -(10)$$

and, from Eq. (4):

$$R^{\lambda}{}_{\mu\nu\rho} = 0 , \quad T^{\lambda}{}_{\mu\nu} = 0 \qquad -(11)$$

reductio ad absurdum.

The curvature tensor may also be obtained from the Cartan identity as follows:

$$R^{\lambda}{}_{\mu\nu\rho} = \partial_\mu T^{\lambda}{}_{\nu\rho} + \partial_\nu T^{\lambda}{}_{\rho\mu} + \Gamma^{\lambda}{}_{\mu\sigma} T^{\sigma}{}_{\nu\rho} + \Gamma^{\lambda}{}_{\nu\sigma} T^{\sigma}{}_{\rho\mu}$$

$$+ \partial_\mu \Gamma^{\lambda}{}_{\rho\nu} - \partial_\nu \Gamma^{\lambda}{}_{\rho\mu} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\rho\nu} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\rho\mu}$$

$$= \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho} \quad -(12)$$

in which the connection may be asymmetric in general. If the connection is assumed to be symmetric then Eq. (12) reduces to:

$$R^\lambda_{\mu\nu\rho} = R^\lambda_{\mu\nu\rho} \quad -(13)$$

because the torsion terms vanish. For a symmetric connection the only possible definition of curvature is, from Eq. (13):

$$R^\lambda_{\mu\nu\rho} = \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho} . \quad -(14)$$

This result implies that the curvature cannot be defined by Eq. (3). This is a reduction to absurdity because Eq. (3) is a mathematical solution of Eq. (2), in which the connection is in general asymmetric.

From Eqs. (3) and (6):

$$\partial_\mu T^\lambda_{\nu\rho} + \Gamma^\lambda_{\mu\sigma} T^\sigma_{\nu\rho} = \frac{1}{2} \left(\partial_\mu T^\lambda_{\nu\rho} + \Gamma^\lambda_{\mu\sigma} T^\sigma_{\nu\rho} - \left(\partial_\nu T^\lambda_{\mu\rho} + \Gamma^\lambda_{\nu\sigma} T^\sigma_{\mu\rho} \right) \right) \quad -(15)$$

so:

$$\partial_\mu T^\lambda_{\nu\rho} + \Gamma^\lambda_{\mu\sigma} T^\sigma_{\nu\rho} = - \left(\partial_\nu T^\lambda_{\mu\rho} + \Gamma^\lambda_{\nu\sigma} T^\sigma_{\mu\rho} \right) \quad -(16)$$

or in equivalent differential form notation:

$$\partial_\mu T^a_{\nu\rho} + \omega^a_{\mu b} T^b_{\nu\rho} = - \left(\partial_\nu T^a_{\mu\rho} + \omega^a_{\nu b} T^b_{\mu\rho} \right) . \quad -(17)$$

This is a new result of differential geometry which again demonstrates antisymmetry under exchange of μ and ν . In ECE theory Eq. (17) is an equation both of gravitation and of electrodynamics. The solution:

$$\partial_\mu T^\lambda_{\ \nu\rho} + \Gamma^\lambda_{\mu\sigma} T^\sigma_{\ \nu\rho} = R^\lambda_{\ \mu\nu\rho} \quad -(18)$$

is the Evans identity, first derived using Hodge duals {1 - 10}. This result is shown as follows. First raise indices:

$$\partial_\mu T^{\lambda\nu\rho} + \Gamma^\lambda_{\mu\sigma} T^{\sigma\nu\rho} = R^\lambda_{\ \mu}{}^{\nu\rho} \quad -(19)$$

so that:

$$D^*_\mu T^{\lambda\nu\rho} = R^\lambda_{\ \mu}{}^{\nu\rho} \quad -(20)$$

and consider the case:

$$\mu = \nu \quad -(21)$$

then:

$$D^*_\mu T^{\lambda\mu\rho} = R^\lambda_{\ \mu}{}^{\mu\rho} \quad -(22)$$

which is the Evans identity, Q. E. D. Eq, (22) is true if repeated indices are summed.

The above proofs are corroborated by proving that:

$$\frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\lambda}{\partial x^{\lambda'}} \frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\lambda}\right) = 0. \quad -(23)$$

The above is the inhomogeneous term in the transformation of the Christoffel connection, as is well known {1 - 10, 12}. Consider the connection defined by:

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_{\ \lambda}. \quad -(24)$$

Its transformation contains the following inhomogeneous term:

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{a}}{\partial x^{a'}} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{a}} \right) = 0 . \quad -(25)$$

To prove this result it is proven that:

$$\frac{\partial x^{\nu'}}{\partial x^{a}} = 0 . \quad -(26)$$

To prove Eq. (26) note that:

$$\frac{\partial x^{\nu'}}{\partial x^{a}} = \frac{\partial x^{\nu'}}{\partial x^{b}} \frac{\partial x^{b}}{\partial x^{a}} . \quad -(27)$$

The tangent spacetime of Cartan's differential geometry is a Minkowski spacetime as is well known {1 - 10, 12}. Therefore by orthogonality:

$$\frac{\partial x^{b}}{\partial x^{a}} = 0 \quad -(28)$$

and Eq. (25) follows. Q.E.D. Therefore the inhomogeneous term in the transformation of $\Gamma^{a}_{\mu\nu}$ vanishes. The inhomogeneous term is however symmetric in μ and a, because partial derivatives with these indices commute. It follows that:

$$\Gamma^{a'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \Gamma^{a}_{\mu\nu} \quad -(29)$$

because it can have no symmetric component in any frame of reference, Q. E. D. From Eq. (24) it follows that:

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \Gamma^{\lambda}_{\mu\nu} . \quad -(30)$$

These proofs each show that the Christoffel connection is antisymmetric and that the EGR is refuted at its most fundamental level.

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