CARTAN GEOMETRY OF PLANE POLAR COORDINATES: PROPERTIES OF THE ACCELERATION AND EULER BERNOULLI RESONANCE.

by

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ABSTRACT

Fundamental classical kinematics are expressed in terms of Cartan geometry and a number of new results found self consistently. The kinetic, potential and total energies are given in general kinematics, together with the collected kinematics of the hyperbolic and logarithmic trajectories, thus describing the main features of whirlpool galaxies. Frenet analysis is applied to find that the Frenet curvature of all planar orbits is the ratio of the linear and angular velocities, and that the Frenet torsion vanishes for all planar orbits. The acceleration due to gravity is expressed in plane polar coordinates and the properties discussed of the centripetal acceleration. In general the acceleration is a Cartan covariant derivative whose spin connection is the angular velocity vector. This is true for the general vector. It is shown that the acceleration expressed in plane polar coordinates produces Euler Bernoulli resonance when a driving force is present. This result can be translated into circuit theory and applied to the theory of energy from spacetime.

Keywords: ECE theory, kinematics, whirlpool galaxies, orbital theory, properties of the acceleration, Euler Bernoulli resonance, circuit theory, energy from spacetime.

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1. INTRODUCTION

In recent papers of this series {1 - 10} the ECE theory has been applied to fundamental classical kinematics, and the angular velocity shown to be the Cartan spin connection. The plane polar system of coordinates is shown in this paper to produce fundamental physics that does not exist in the Cartesian system. Examples are the Coriolis and centripetal accelerations, and Euler Bernoulli resonance. The reason is that the plane polar coordinates are rotating, while the Cartesian coordinates are static. This paper as usual summarizes extensive calculations in the accompanying background notes to UFT237 on www.aias.us. In Section 2, fundamental kinematics are developed systematically to give expressions for the velocity, kinetic energy, potential energy and total energy or hamiltonian. The results are applied to show that the main features of the whirlpool galaxy can be explained with a hyperbolic spiral orbit but not with a logarithmic spiral orbit. The complete kinematics of both types of orbits are given in the background notes, together with material on various types of Cotes and Poinsot spirals. The kinematics are developed with the Frenet {11} system of coordinates and it is shown that the Frenet curvature is the ratio of the linear to angular velocities for all planar orbits. It is shown that the Frenet torsion vanishes for all orbits in a plane. The Frenet curvature and torsion must not be confused with the Cartan curvature and torsion. It is emphasized that the acceleration due to gravity produces the Coriolis and centripetal terms in the plane polar system. The description of an elliptical orbit for example depends on which coordinate system is used. In general the velocity and acceleration in plane polar coordinates are covariant derivatives of Cartan, and are worked out in all detail and self consistently. Finally in Section 2 it is shown that Euler Bernoulli resonance can be induced in the plane polar coordinates when it is absent in the Cartesian system of coordinates.

In Section 3 the results for Euler Bernoulli resonance are translated into circuit theory, a circuit may be constructed in which the driving force originates in spacetime. SO energy may be obtained from spacetime.

2. DEVELOPMENT OF KINEMATICS

The covariant derivative of Cartan may be defined for use in classical kinematics in space. For any vector $\underline{V}$ the covariant derivative is:

$$\frac{D\underline{V}}{dt} = \left(\frac{d\underline{V}}{dt}\right)_{\text{axes fixed}} + \underline{\omega} \times \underline{V} \quad -(1)$$

where the spin connection vector is the angular velocity vector $\underline{\omega}$. In plane polar coordinates define:

$$\underline{V} = V\underline{e}_r \quad -(2)$$

for simplicity of development. The velocity is then defined as:

$$\underline{v} = \frac{D\underline{r}}{dt} = \frac{d\underline{r}}{dt} + \underline{\omega} \times \underline{r} \quad -(3)$$

where:

$$\frac{d\underline{r}}{dt} := \left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} \quad -(4)$$

By definition {11, 12}:

$$\frac{D\underline{r}}{dt} = \frac{D}{dt}\left(r\underline{e}_r\right) = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt} \quad -(5)$$

so

$$\left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} = \frac{dr}{dt}\underline{e}_r \quad -(6)$$

and:

$$\underline{\omega} \times \underline{r} = r\frac{d\underline{e}_r}{dt}. \quad -(7)$$

The acceleration is defined as:

$$\underline{a} = \frac{D\underline{v}}{dt} = \frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} \quad -(8)$$

where:

$$\frac{d\underline{v}}{dt} := \left(\frac{d\underline{v}}{dt}\right)_{\text{axes fixed}}. \quad -(9)$$

From fundamental kinematics as in the preceding papers in this series:

$$\underline{a} = \frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} = \left(\ddot{r} - \omega^2 r\right)\underline{e}_r + \left(r\frac{d\omega}{dt} + 2\frac{dr}{dt}\omega\right)\underline{e}_\theta$$

$$= \frac{d^2r}{dt^2}\underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) + r\frac{d\omega}{dt}\underline{e}_\theta + 2\frac{dr}{dt}\omega\,\underline{e}_\theta \quad -(10)$$

where $\underline{e}_r$ and $\underline{e}_\theta$ are the unit vectors {11} of the plane polar system of coordinates:

$$\begin{aligned} \underline{e}_r \times \underline{e}_\theta &= \underline{k} \\ \underline{k} \times \underline{e}_r &= \underline{e}_\theta \\ \underline{e}_\theta \times \underline{k} &= \underline{e}_r. \end{aligned} \quad -(11)$$

Therefore:

$$\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} = \frac{d^2r}{dt^2}\underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) + r\frac{d\omega}{dt}\left(\underline{k} \times \underline{e}_r\right) + 2\frac{dr}{dt}\omega\left(\underline{k} \times \underline{e}_r\right)$$

$$= \frac{d^2r}{dt^2}\underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) + \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \left(\frac{dr}{dt}\underline{e}_r\right). \quad -(12)$$

From Eq. (3):

$$\underline{v} = \frac{d\underline{r}}{dt} + \underline{\omega} \times \underline{r} \quad -(13)$$

so in Eq. (8):

$$\underline{a} = \frac{d}{dt}\left(\frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r}\right)_{\substack{\text{axes}\\ \text{fixed}}} + \underline{\omega} \times \left(\frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r}\right)$$

$$= \frac{d^2 r}{dt^2}\underline{e}_r + \frac{d\underline{\omega}}{dt}\times\underline{r} + \underline{\omega}\times\left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} + \underline{\omega}\times\frac{dr}{dt}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}). \quad -(14)$$

In this equation:

$$\underline{\omega}\times\left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} = \underline{\omega}\times\frac{dr}{dt}\underline{e}_r \quad -(15)$$

so:

$$\underline{a} = \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\left(\frac{dr}{dt}\underline{e}_r\right) \quad -(16)$$

which is Eq. (12), QED. The covariant derivatives used in these calculations are examples of the Cartan covariant derivative {1 - 10}

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(17)$$

It is seen that the well known {12} centripetal acceleration:

$$\underline{a}_{\text{centripetal}} = \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(18)$$

and Coriolis acceleration:

$$\underline{a}_{\text{Coriolis}} = \frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\left(\frac{dr}{dt}\underline{e}_r\right) \quad -(19)$$

are produced by the plane polar system of coordinates. These accelerations do not exist in the Cartesian system and depend entirely on the existence of the spin connection of Cartan. The entire theory of classical rotational motion depends on the spin connection. In preceding papers it was shown that the Coriolis acceleration vanishes for all closed orbits in a plane. These can orbits in astronomy or on a laboratory bench. In this case the acceleration in the plane polar coordinate system simplifies to:

$$\underline{a} = (\ddot{r} - \omega^2 r)\underline{e}_r = \left(\frac{d^2 r}{dt^2} - \omega^2 r\right)\underline{e}_r$$
$$= \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}). \quad -(20)$$

For example {12} the acceleration due to gravity in the plane polar system is:

$$\underline{g} = \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(21)$$

and includes the centripetal acceleration:

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r \underline{e}_r. \quad -(22)$$

Therefore $\underline{g}$ is not normally directed to the earth's surface because of the centripetal acceleration due to the earth's angular velocity. The $\underline{g}$ in the plane polar system is the sum of the $\underline{g}$ in the Cartesian system:

$$\underline{g}\,(\text{Cartesian}) = \frac{d^2 r}{dt^2}\underline{e}_r \quad -(23)$$

and the centripetal acceleration. To make this point clearer consider the acceleration of an elliptical orbit or closed elliptical trajectory in the plane polar system. It has been shown in previous work that it is:

$$\underline{a} = -\frac{L^2}{m^2 r^2 \alpha}\underline{e}_r \quad -(24)$$

where the angular momentum is a constant of motion and is defined by:

$$L = |\underline{L}| = |\underline{r} \times \underline{p}| = m r^2 \omega. \quad -(25)$$

Here m is the mass of an object moving along the orbit or trajectory and r is the distance between one focus of the ellipse and the object. The symbol α denotes the half right latitude or semi latus rectum of the ellipse. So the acceleration due to gravity generated by the elliptical motion of the mass m is:

$$\underline{g} = -\frac{L^2}{m^2 r^2 \alpha} \underline{e}_r \quad -(26)$$

in plane polar coordinates. To recover the Newtonian result {12}:

$$\alpha = \frac{L^2}{m^2 M G} \quad -(27)$$

where M is the mass of an object at the focus of the ellipse and G is Newton's constant. With the assertion (27) Eq. (26) becomes:

$$\underline{g} = -\frac{MG}{r^2} \underline{e}_r . \quad -(28)$$

The force is defined from Eq. (28) as:

$$\underline{F} = m \underline{g} = -\frac{mMG}{r^2} \underline{e}_r . \quad -(29)$$

This is the only force present in the plane polar system of coordinates.

The acceleration in the Cartesian system of coordinates from Eq. (21) is:

$$\underline{a}(\text{Cartesian}) = \underline{g} - \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(30)$$

in which the centrifugal acceleration is:

$$-\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \omega^2 r \underline{e}_r . \quad -(31)$$

Therefore in the Cartesian system the acceleration produced by the same elliptical trajectory is:

$$\left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r = \left(-\frac{L^2}{m^2 r^2 \alpha} + \omega^2 r\right) \underline{e}_r \quad -(32)$$

and this is a generally valid kinematic result. It generalizes the Newtonian theory, which is again obtained from Eq. (27) to give:

$$\left(\frac{d^2 r}{dt^2}\right)_{Cartesian} \underline{e}_r = \left(-\frac{MG}{r^2} + \omega^2 r\right) \underline{e}_r = \left(-\frac{MG}{r^2} + \frac{L^2}{m^2 r^3}\right) \underline{e}_r \quad -(33)$$

and the familiar force:

$$\underline{F} = m \left(\frac{d^2 r}{dt^2}\right)_{Cartesian} \underline{e}_r = \left(-\frac{mMG}{r^2} + \frac{L^2}{m r^3}\right) \underline{e}_r \quad -(34)$$

of the textbooks {12}. From a comparison of Eqs. (29) and (34) the forces in the plane polar and Cartesian coordinate systems are different. If the frame of reference is static with respect to the observer, the force is defined by Eq. (34). If the frame of reference is rotating with respect to the observer, the force is defined by Eq.(29).

The easiest way to approach this analysis is always to calculate the acceleration first in plane polar coordinates, and to realize that one term of the resultant expression is the acceleration in the Cartesian system. For an observer on the earth orbiting the sun, the relevant expression is that in the Cartesian frame, because the latter is also fixed on the earth and does not move with respect to the observer. In other words the observer is in his own frame of reference. For an observer on the sun, the relevant expression is that in the plane polar system of coordinates, because the earth rotates with respect to the observer.

The observer on the earth experiences the centrifugal acceleration:

$$-\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \omega^2 r \, \underline{e}_r \quad -(35)$$

directed outwards from the earth. This is the origin of the everyday centrifugal force. The observer on the sun experiences the centripetal acceleration:

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r \, \underline{e}_r \quad -(36)$$

directed towards the sun and towards the observer. The entire analysis rests on the spin connection and on the fact that in the plane polar system the frame itself is rotating and thus generates the spin connection by definition.

It was shown in preceding papers that for any planar orbit the force in the plane polar coordinates is:

$$\underline{F} = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right)\underline{e}_r \quad -(37)$$

a result which is obtained purely from kinematics and is more general than the Newtonian theory. The result (37) is consistent with Lagrangian theory {12} and includes the centripetal acceleration, which appears as the second term on the right hand side of Eq. (37). Eq. (37) is very useful because it gives the force law for any planar and closed orbit, or planar and closed trajectory on a laboratory bench. A similar equation can be obtained for the velocity. In plane polar coordinates the velocity is {11, 12}:

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \omega r \underline{e}_\theta = \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} . \quad -(38)$$

Using the chain rule:

$$\frac{dr}{d\theta} = -r^2\frac{d}{d\theta}\left(\frac{1}{r}\right) = \frac{dr}{dt}\frac{dt}{d\theta}, \quad -(39)$$

$$\frac{dr}{dt} = -\omega r^2\frac{d}{d\theta}\left(\frac{1}{r}\right), \quad -(40)$$

it follows that the velocity in plane polar coordinates can be expressed as

$$\underline{v} = -\omega r^2\frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r + \omega r\underline{e}_\theta = \left(\frac{L}{m}\right)\left(\frac{1}{r}\underline{e}_\theta - \frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r\right) \quad -(41)$$

for any planar orbit. The kinetic energy of any planar orbit is therefore:

$$T = \frac{1}{2} m v^2 = \frac{L^2}{2m} \left(\frac{1}{r^2} + \left(\frac{d}{d\theta} \left(\frac{1}{r} \right) \right)^2 \right) \quad -(42)$$

The potential energy is defined by the force {12}:

$$F = - \frac{\partial U}{\partial r} \quad -(43)$$

so:

$$U = \int \frac{L^2}{m r^2} \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) dr \quad -(44)$$

The kinetic energy is defined in general by the work integral {12}:

$$W_{12} = \int_1^2 \underline{F} \cdot d\underline{r} = T_2 - T_1 \quad -(45)$$

and if

$$T_1 = 0 \quad -(46)$$

then

$$W = T = \left(\int \underline{F} \cdot d\underline{r} \right)_{kinetic} \quad -(47)$$

So the total energy is:

$$E = H = \left(\int \underline{F} \cdot d\underline{r} \right)_{kinetic} - \left(\int F dr \right)_{potential} = \text{constant.} \quad -(48)$$

In other words the total energy is due entirely to integrals over the force and therefore over the acceleration. Again in this context the fundamental kinematics are more general than the Newtonian theory. The kinetic energy is therefore:

$$T = W = \int \frac{d}{dt} \left(m \underline{v} \right) \cdot d\underline{r} \quad -(49)$$

where
$$d\underline{r} = \underline{v}\, dt \quad -(50)$$

so we obtain the familiar result:

$$T = W = \int \left(\frac{d}{dt} m\underline{v} \right) \cdot \underline{v}\, dt = \frac{m}{2} \int \frac{d}{dt} (\underline{v} \cdot \underline{v})\, dt = \frac{1}{2} m v^2 . \quad -(51)$$

The potential energy is defined as:

$$W = U_1 - U_2 = \int_1^2 \underline{F} \cdot d\underline{r} \quad -(52)$$

and is the work done {12} in moving a mass m from 1 to 2. Eq. (52) implies:

$$\underline{F} = -\underline{\nabla} U \quad -(53)$$

so:

$$\int_1^2 \underline{F} \cdot d\underline{r} = -\int_1^2 (\underline{\nabla} U) \cdot d\underline{r} = -\int_1^2 dU = U_1 - U_2 \quad -(54)$$

QED. It is seen that the fundamental definitions of kinetic and potential energy result from the equivalence principle:

$$\underline{F} = m \frac{d\underline{v}}{dt} = -\underline{\nabla} U \quad -(55)$$

which in previous work was derived and proven from the antisymmetry laws of ECE theory.

So in general kinematics:

$$H = E = \left(\int \underline{F} \cdot d\underline{r} \right)_{kinetic} + \left(\int \underline{F} \cdot d\underline{r} \right)_{potential} \quad -(56)$$

where:

$$\left(\int \underline{F} \cdot d\underline{r} \right)_{kinetic} = \int m \frac{d\underline{v}}{dt} \cdot d\underline{r} \quad -(57)$$

and
$$\left(\int \underline{F}\cdot d\underline{r}\right)_{potential} = -\int \underline{\nabla}\, U \cdot d\underline{r} \qquad -(58)$$

In the case of a closed elliptical orbit:
$$H = E = \frac{1}{2}m\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right) - \frac{L^2}{m\alpha r} \qquad -(59)$$

an equation which again is the result of fundamental kinematics. It can be shown as follows that Eq. (59) for the total energy or hamiltonian is the equation of an ellipse. First use the chain rule:
$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega\frac{dr}{d\theta} \qquad -(60)$$

and the definition:
$$L = m\omega^2 r \qquad -(61)$$

to find that:
$$E = \frac{L^2}{2mr^4}\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) - \frac{L^2}{m\alpha r} \qquad -(62)$$

The ellipse in plane polar coordinates is:
$$r = \frac{\alpha}{1+\epsilon\cos\theta} \qquad -(63)$$

where ϵ is the ellipticity {12}. From Eq. (63):
$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon r^2}{\alpha}\right)^2\sin^2\theta = \left(\frac{\epsilon r^2}{\alpha}\right)^2\left(1-\frac{1}{\epsilon^2}\left(\frac{\alpha}{r}-1\right)^2\right)$$

$$= \frac{r^4}{\alpha^2}\left(\epsilon^2 - 1\right) + 2r^2\left(\frac{r}{\alpha}\right) - r^2 . \quad -(64)$$

Using Eq. (64) in Eq. (62):

$$E = \frac{L^2}{m}\left[\frac{1}{2r^4}\left(\frac{r^4}{\alpha^2}\left(\epsilon^2 - 1\right) + \frac{2r^3}{\alpha}\right) - \frac{1}{\alpha r}\right]$$

$$= \left(\epsilon^2 - 1\right)\frac{L^2}{2m\alpha^2} \quad -(65)$$

so Eq. (59) and Eq. (63) are both ellipses if

$$\frac{\epsilon^2 - 1}{\alpha^2} = \frac{2mE}{L} . \quad -(66)$$

The total energy is therefore:

$$H = E = T + U = \frac{L}{2m}\left(\frac{\epsilon^2 - 1}{\alpha^2}\right) \quad -(67)$$

and is a constant of motion along with the total angular momentum L. Note carefully that the result (66) is more general than the Newtonian theory, and is valid for all conical sections.

(The accompanying notes 237(3) and 237(4) posted on www.aias.us with UFT237 illustrate this point by developing the complete kinematics of the hyperbolic and logarithmic spirals using the above equations. In this way it is easily shown that the main features of the whirlpool galaxy can be described with stars emerging from the centre on a hyperbolic spiral. The stars reach a constant velocity at infinite r - the well known velocity curve. Both Newtonian and Einsteinian theory fail qualitatively to describe this result as is well known. The fundamental kinematics of this section describe it self consistently and elegantly. On the other hand stars on a logarithmic spiral or other type of spiral cannot explain the observed velocity curve at all. Notes 237(5) describe Cotes and Poinsot spirals for example, and it is found that the precessing ellipse is a type of Poinsot spiral added to a constant.

The above analysis may be developed with the well known {11} Frenet equations of differential geometry to give new insights. In the Frenet analysis a curve is parameterized as follows:

$$\underline{r} = \underline{r}(s). \quad -(68)$$

The tangent and normal unit vectors are defined as {11}:

$$\underline{T} = \frac{d\underline{r}}{ds}, \quad \underline{N} = \rho \frac{d\underline{T}}{ds} \quad -(69)$$

where ρ is the radius of curvature. In the plane polar coordinates {11}:

$$\underline{r} = r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j}. \quad -(70)$$

Now define:

$$s = r \quad -(71)$$

so:

$$\underline{T} = \frac{d\underline{r}}{dr} = \cos\theta\, \underline{i} + \sin\theta\, \underline{j} = \underline{e}_r. \quad -(72)$$

The normal unit vector is defined as:

$$\underline{N} = \rho \frac{d\underline{T}}{dr} = \rho \frac{d\underline{T}}{d\theta}\frac{d\theta}{dr} = \rho \frac{d\theta}{dr}\underline{e}_\theta. \quad -(73)$$

However, $\underline{N}$ and $\underline{e}_\theta$ are unit vectors, so the Frenet curvature in the plane polar coordinates is:

$$\rho = \frac{dr}{d\theta} \quad -(74)$$

We arrive at the important result that the Frenet curvature of any planar orbit can be found from Eq. (74). In addition it is found that:

$$\underline{T} = \underline{e}_r \,, \quad \underline{N} = \underline{e}_\theta \,. \quad -(75)$$

The velocity is defined by:

$$\begin{aligned} \underline{v} &= v\,\underline{e}_r + r\omega\,\underline{e}_\theta \\ &= v\,\underline{T} + r\frac{d\theta}{dr}\frac{dr}{dt}\,\underline{N} \\ &= v\,\underline{T} + \frac{vr}{\rho}\,\underline{N} \end{aligned} \quad -(76)$$

The binormal unit vector of Frenet is defined in the plane polar coordinates by:

$$\underline{B} = \underline{T} \times \underline{N} = \underline{e}_r \times \underline{e}_\theta = \underline{k} \,. \quad -(77)$$

The third Frenet formula {11} is:

$$\frac{d\underline{N}}{ds} = \tau\,\underline{B} - \frac{1}{\rho}\,\underline{T} \quad -(78)$$

where τ is the Frenet torsion. So:

$$\frac{d\underline{e}_\theta}{dr} = \tau\,\underline{k} - \frac{1}{\rho}\,\underline{e}_r = \frac{d\underline{e}_\theta}{dt}\frac{dt}{dr} = -\frac{\omega}{v}\,\underline{e}_r \,. \quad -(79)$$

It follows that:

$$\rho = \frac{v}{\omega} \,, \quad \tau = 0 \quad -(80)$$

for all planar orbits. For a circular orbit:

$$\rho = r \,. \quad -(81)$$

The same result can be obtained using the chain rule as follows:

$$\omega = \frac{d\theta}{dr}\frac{dr}{dt} = \frac{v}{\rho} \,. \quad -(82)$$

The acceleration in the plane polar coordinates is:

$$\underline{a} = \left(\ddot{r} - r\omega^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta \quad -(83)$$

an expression which can eb developed using the definition in the Frenet analysis of the tangential and normal components of velocity:

$$\underline{V} = v_T\,\underline{T} + v_N\,\underline{N} \quad -(84)$$

i.e.

$$v_T = \frac{dr}{dt}, \quad v_N = \omega r. \quad -(85)$$

The tangential acceleration is:

$$\underline{a}_T = \ddot{r}\,\underline{e}_r + \dot{r}\dot{\theta}\,\underline{e}_\theta = \frac{dv}{dt}\underline{T} + \frac{v^2}{\rho}\underline{N} \quad -(86)$$
$$= d\left(v_T\underline{T}\right)/dt$$

and the normal acceleration is:

$$\underline{a}_N = \frac{d}{dt}\left(\frac{vr}{\rho}\underline{N}\right) = -\omega^2 r\,\underline{T} + \frac{d}{dt}(\omega r)\,\underline{N}. \quad -(87)$$

Note that:

$$\frac{d}{dt}\left(\frac{v}{\rho}\right) = \frac{d\omega}{dt} = \frac{1}{\rho}\frac{dv}{dt} - \frac{v}{\rho^2}\frac{d\rho}{dt} \quad -(88)$$

so:

$$\frac{d}{dt}(\omega r) = \omega\frac{dr}{dt} + \frac{r}{\rho}\frac{dv}{dt} - \frac{rv}{\rho^2}\frac{d\rho}{dt} \quad -(89)$$

for any orbit in a plane. Therefore:

$$\underline{a}_T = \ddot{r}\,\underline{T} + \dot{r}\dot{\theta}\,\underline{N} = \ddot{r}\,\underline{e}_r + \dot{r}\dot{\theta}\,\underline{e}_\theta \quad -(90)$$

$$\underline{a}_N = -\omega^2 r\,\underline{T} + \left(r\ddot{\theta} + \dot{\theta}\dot{r}\right)\underline{N} \quad -(91)$$
$$= -\omega^2 r\,\underline{e}_r + \left(r\ddot{\theta} + \dot{\theta}\dot{r}\right)\underline{e}_\theta$$

The acceleration in the Frenet system is therefore:

$$\underline{a} = \underline{a}_T + \underline{a}_N = \ddot{r}\,\underline{T} - \omega^2 r\,\underline{T} + \left(\ddot{\theta} r + 2\dot{\theta}\dot{r}\right)\underline{N}. \quad -(92)$$

The Cartesian term in this expression is:

$$\underline{a}\,(\text{Cartesian}) = \ddot{r}\,\underline{T}. \quad -(93)$$

The centripetal acceleration is:

$$\underline{a}\,(\text{centripetal}) = -\omega^2 r\,\underline{T}, \quad -(94)$$

and the Coriolis acceleration is:

$$\underline{a}\,(\text{Coriolis}) = \left(2\dot{\theta}\dot{r} + \ddot{\theta} r\right)\underline{N}. \quad -(95)$$

Finally in this Section consider again the acceleration in plane polar coordinates in the absence of the Coriolis acceleration:

$$\frac{d^2\underline{r}}{dt^2} = \left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) \quad -(96)$$

and example bing a closed orbit or trajectory in a plane. The acceleration in Cartesian coordinates is:

$$\left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r = \frac{d^2\underline{r}}{dt^2} + \omega^2 r\,\underline{e}_r = \frac{d^2\underline{r}}{dt^2} + \omega^2\underline{r}. \quad -(97)$$

Therefore we arrive at the important result:

$$\left(\frac{d^2\underline{r}}{dt^2}\right)_{\text{Cartesian}} = \left(\frac{d^2\underline{r}}{dt^2} + \omega^2\underline{r}\right)_{\text{plane polar}} \quad -(98)$$

Eq. (98) becomes an Euler Bernoulli resonance equation if:

$$\frac{d^2 \underline{r}}{dt^2} + \omega^2 \underline{r} = A \cos \omega_1 t \, \underline{e}_r \quad -(99)$$

where the right hand side describes a driving term in plane polar coordinates. By definition:

$$\underset{\sim}{r} = X \underline{i} + Y \underline{j} = r \cos\theta \, \underline{i} + r \sin\theta \, \underline{j} \quad -(100)$$

and {11}:

$$\underline{e}_r = \cos\theta \, \underline{i} + \sin\theta \, \underline{j} \, . \quad -(101)$$

So equating vector components of Eq. (99)

$$\frac{d^2}{dt^2}\left(r\cos\theta\right) + \omega^2 r \cos\theta = A \cos\omega_1 t \cos\theta \quad -(102)$$

and

$$\frac{d^2}{dt^2}\left(r\sin\theta\right) + \omega^2 r \sin\theta = A \cos\omega_1 t \sin\theta \quad -(103)$$

i.e.:

$$\frac{d^2 X}{dt^2} + \omega^2 X = A_X \cos\omega_1 t \quad -(104)$$

and

$$\frac{d^2 Y}{dt^2} + \omega^2 Y = A_Y \cos\omega_1 t \quad -(105)$$

These are Euler Bernoulli resonance equations with solutions {12}:

$$X(t) = \frac{A_X \cos\omega_1 t}{\omega^2 - \omega_1^2} \quad -(106)$$

and:

$$Y(t) = \frac{A_Y \cos \omega_1 t}{\omega^2 - \omega_1^2} \quad -(107)$$

The resonance condition is:

$$\omega = \omega_1 \quad -(108)$$

at which point:

$$\underset{\sim}{r} \rightarrow \infty . \quad -(109)$$

The resonance is present in the plane polar system but not present in the Cartesian system. Similarly for the Coriolis and centripetal accelerations. Therefore important new physics emerges from the plane polar coordinates. In the next section this result is translated into circuit theory.

3. CIRCUIT DESIGN FROM EQS. (104) and (105).

Section by Dr. Horst Eckardt.

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