

ECE / BELTRAMI THEORY OF THE PARTON STRUCTURE OF
ELEMENTARY PARTICLES.

by

M. W. Evans and H. Eckardt,
Civil List, AIAS and UPITEC

ABSTRACT

The parton structure of elementary particles such as the electron, proton and neutron is developed using a constrained Schrodinger equation deduced from the Beltrami equation for linear momentum. The constraint is a direct result of the geometry of the Beltrami equation, which is an example of the geometrically based ECE theory. Using the conservation of total energy E , a differential equation is obtained in potential energy V , which is used in the Schrodinger equation to evaluate the energy levels and wavefunctions of the elementary particle. Partons are the result of the wavefunctions.

Keywords: ECE Theory, Beltrami theory, parton structure of elementary particles.

UFT 260



1. INTRODUCTION

Recent papers in this series {1 - 10} have produced a vectorial format of the Cartan identity and a geometrical theory of charge current density. The Beltrami formalism has been incorporated into ECE theory and in Section 2 of this paper the Beltrami equation for linear momentum \underline{p} is considered. Using the quantum postulate the Beltrami equation is developed into a Schroedinger equation with a constraint, a cubic equation in $V - E$, where V is the potential energy and where E is the total energy. Using the conservation of total energy E , this equation is reduced to a differential equation in V , and V is used in the Schroedinger equation to find the wavefunctions and energy levels of the interior structure of an elementary particle such as an electron, proton or neutron. Therefore it is assumed that the interior structure of an elementary particle is governed by a Beltrami equation in linear momentum \underline{p} . This is the starting hypothesis, chosen because the Beltrami equation leads to the powerful Schroedinger equation or inhomogeneous Helmholtz equation. In this series of two hundred and sixty papers to date, nearly all the precepts of the standard model have been refuted, so the quark gluon model is rejected on the grounds that it is a meaningless curve fitting exercise based on nineteen adjustables and many errors and obscurities {1 - 10}. Background notes one to five accompanying UFT260 on www.aias.us lead in to Section 2, and should be read as part of the paper. These notes deal with absence of curvature in the vacuum, the Beltrami structure of various quantities in the absence of a magnetic monopole, the conditions under which the tetrad is a Beltrami function, spherical solutions of the Helmholtz equation and the basic equation of matter in terms of the tetrad. These notes deal with the homogeneous Helmholtz equation, but in order to develop a parton theory the Schroedinger equation is needed. The Beltrami condition on linear momentum constrains the Schroedinger equation in such a way as to produce a rich internal structure, containing as much information as the

quark model, but with only one parameter compared with nineteen, and with complete absence of obscurity.

2 DERIVATION OF THE CONSTRAINED SCHROEDINGER EQUATION

Consider the Beltrami equation in linear momentum:

$$\underline{\nabla} \times \underline{p} = \kappa \underline{p} \quad - (1)$$

where in general κ depends on coordinates and is not a constant. From Eq. (1)

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{p}) = \underline{\nabla} \times (\kappa \underline{p}) \quad - (2)$$

By vector analysis Eq. (2) can be developed as:

$$\underline{\nabla} (\underline{\nabla} \cdot \underline{p}) - \nabla^2 \underline{p} = \kappa^2 \underline{p} + \underline{\nabla} \kappa \times \underline{p} \quad - (3)$$

so:

$$(\nabla^2 + \kappa^2) \underline{p} = \underline{\nabla} (\underline{\nabla} \cdot \underline{p}) - \underline{\nabla} \kappa \times \underline{p}. \quad - (4)$$

One possible solution is:

$$(\nabla^2 + \kappa^2) \underline{p} = \underline{0} \quad - (5)$$

and:

$$\underline{\nabla} (\underline{\nabla} \cdot \underline{p}) = \underline{\nabla} \kappa \times \underline{p}. \quad - (6)$$

Eq. (6) implies:

$$\underline{p} \cdot \underline{\nabla} (\underline{\nabla} \cdot \underline{p}) = \underline{p} \cdot \underline{\nabla}^2 \underline{p} = 0. \quad (7)$$

Two possible solutions of Eq. (7) are:

$$\underline{\nabla} \cdot \underline{p} = 0 \quad (8)$$

and:

$$\underline{\nabla} (\underline{\nabla} \cdot \underline{p}) = 0. \quad (9)$$

Now use the quantum postulate:

$$\underline{p} = -i\hbar \underline{\nabla}. \quad (10)$$

Eqs. (5) and (10) give:

$$(\nabla^2 + \kappa^2) \underline{\nabla} \psi = 0 \quad (11)$$

and the Schroedinger equation is [1-10]:

$$(\nabla^2 + \kappa^2) \psi = 0. \quad (12)$$

From Eq. (12):

$$\underline{\nabla} \left((\nabla^2 + \kappa^2) \psi \right) = 0 \quad (13)$$

i. e.

$$(\nabla^2 + \kappa^2) \underline{\nabla} \psi + \underline{\nabla} \left((\nabla^2 + \kappa^2) \psi \right) = 0 \quad (14)$$

a possible solution of which is:

$$(\nabla^2 + \kappa^2) \underline{\nabla} \psi = 0 \quad (15)$$

and

$$\underline{\nabla} (\underline{\nabla}^2 + \kappa^2) \psi = \underline{0} \quad - (16)$$

Eq. (16) is Eq. (11) Q.E.D. Eq. (16) can be written as:

$$\underline{\nabla} \nabla^2 \psi + \underline{\nabla} \kappa^2 \psi = 0, \quad - (17)$$

i.e.

$$\underline{\nabla} (\nabla^2 \psi + \kappa^2 \psi) = \underline{0}. \quad - (18)$$

A possible solution of Eq. (18) is the Schroedinger equation

$$(\nabla^2 + \kappa^2) \psi = 0. \quad - (19)$$

So the Schroedinger equation is compatible with Eq. (11).

Eq. (8) gives:

$$\nabla^2 \psi = 0 \quad - (20)$$

which is consistent with Eq. (19) only if:

$$\kappa = 0 \quad - (21)$$

Eq. (9) gives:

$$\underline{\nabla} (\nabla^2 \psi) = \underline{0} \quad - (22)$$

where:

$$\nabla^2 \psi = -\kappa^2 \psi. \quad - (23)$$

Therefore:

$$\underline{\nabla} (\kappa^2 \psi) = (\underline{\nabla} \kappa^2) \psi + \kappa^2 \underline{\nabla} \psi = 0 \quad - (24)$$

and:

$$\underline{\nabla} \psi = - \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \right) \psi. \quad (25)$$

Therefore:

$$\begin{aligned} \underline{\nabla} \cdot \underline{\nabla} \psi &= \nabla^2 \psi = - \underline{\nabla} \cdot \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \psi \right) \\ &= - \left(\underline{\nabla} \cdot \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \right) \right) \psi + \left(\frac{\underline{\nabla} \kappa^2}{\kappa^2} \right) \cdot \underline{\nabla} \psi, \end{aligned} \quad (26)$$

from a comparison of Eqs. (19) and (26) we obtain the subsidiary condition:

$$\kappa^2 \nabla^2 \kappa^2 = \underline{\nabla} \kappa^2 \cdot \underline{\nabla} \kappa^2 + \kappa^6 \quad (27)$$

where:

$$\kappa^2 = \frac{2m}{\hbar^2} (V - E). \quad (28)$$

Therefore:

$$\underline{\nabla} \kappa^2 = \frac{2m}{\hbar^2} \underline{\nabla} V \quad (29)$$

and

$$\nabla^2 \kappa^2 = \frac{2m}{\hbar^2} \nabla^2 V \quad (30)$$

giving a cubic constraint equation in $V - E$:

$$(V - E)^3 - \frac{\hbar^2}{2m} \nabla^2 V (V - E) + \frac{\hbar^2}{2m} (\underline{\nabla} V \cdot \underline{\nabla} V) = 0 \quad (31)$$

This can be written as a cubic equation in E, which is a constant. E is expressed in terms of V, ∇V , and $\nabla^2 V$. Using:

$$\nabla E = 0 \quad - (32)$$

gives a differential equation in V which can be solved numerically, giving an expression for V. Finally this expression for V is used in the Schroedinger equation:

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E \psi \quad - (33)$$

to find the energy levels of E and the wavefunctions ψ . These are the energy levels and wavefunctions of the interior parton structure of an elementary particle such as an electron, proton or neutron. The well developed methods of computational quantum mechanics can be used to find the expectation values of any property and can be applied to scattering theory, notably deep inelastic electron electron, electron proton and electron neutron scattering. These data are claimed to provide evidence for quark structure, but the quark model depends on the validity of the U(1) and electroweak sectors of the standard model. In this series of papers these sector theories have been refuted in many ways.

3. COMPUTATIONAL RESULTS

Section by Dr. Horst Eckardt.

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ECE/Beltrami theory of the parton structure of elementary particles

M. W. Evans*[‡]; H. Eckardt[†]
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

2 Corrections to section 2

... we obtain the subsidiary condition:

$$\nabla^2 \kappa^2 = \kappa^4 \quad (27)$$

... giving a quadratic equation in $V - E$:

$$\nabla^2 (V - E) = \frac{2m}{\hbar^2} (V - E)^2 \quad (31)$$

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E \psi \quad (33)$$

3 Numerical solution of the constraint Schroedinger equation

3.1 Solution of the constraint equation (27)

In this section a numerical solution of the constraint Schroedinger equation (33) is developed. The potential is obtained from the constraint equation (27) or (31), respectively. We choose the form (27) for κ^2 which holds for all energies E so a solution of (27) is universal in E . For the electron it is known that there is no angular dependence of the particle charge density. For the proton there is only a weak angular dependence. Therefore we restrict the ∇^2 operator in (27) to the radial part, giving

$$\frac{d^2}{dr^2} \kappa^2(r) + \frac{2}{r} \frac{d}{dr} \kappa^2(r) = \kappa^4(r) \quad (34)$$

*email: emyrone@aol.com

†email: mail@horst-eckardt.de

with

$$\kappa^2 = \frac{2m(V - E)}{\hbar^2} \quad (35)$$

as before. When κ^2 is known, the potential is obtainable by

$$V = E + \frac{\hbar^2 \kappa^2}{2m}. \quad (36)$$

In order to simplify Eq.(34) we substitute κ by a new function λ :

$$\lambda^2(r) := r \kappa^2(r). \quad (37)$$

This is the same procedure as getting rid of the first derivative in the standard solution procedure for the radial Schroedinger equation. Eq.(34) then reads:

$$\frac{d^2}{dr^2} \lambda^2(r) = \frac{\lambda^4(r)}{r}. \quad (38)$$

The initial conditions have to be chosen as follows. Because the radial coordinate in (37) starts at $r = 0$, we have to use $\lambda^2(0) = 0$ to be consistent. For the derivative of λ^2 follows from (37):

$$\frac{d\lambda^2}{dr} = \kappa^2 + 2r \frac{d\kappa}{dr}. \quad (39)$$

Only the first term contributes for $r = 0$ so that the initial value of κ^2 determines the derivative of λ^2 at this point. In total:

$$\lambda^2(0) = 0, \quad (40)$$

$$\frac{d\lambda^2}{dr}(0) = \kappa^2(0). \quad (41)$$

If $\kappa^2(0)$ is positive, we obtain only functions with positive curvature for λ^2 and κ^2 , see Fig. 1. The potential function is always positive and greater than zero, allowing no bound states. Both functions diverge for large r . Therefore we have to start with a negative value of $\kappa^2(0)$. Then we obtain a negative region of the potential function, beginning with a horizontal tangent. This is the same as in the Woods Saxon potential, a model potential for of atomic nuclei. There is no singularity at the origin because there is no point charge.

Numerical studies give the result that the solutions λ^2 and κ^2 are always of the type shown in Fig. 2. The radial scale is determined by the depth of the initial value $\kappa^2(0)$. We have chosen this value so large that the radial scale (in atomic units) is in the range of the radii of elementary particles, see Table 1. As an artifact, the diverging behaviour for $r \rightarrow \infty$ found previously remains for negative initial values of the potential function. Obviously κ^2 crosses zero when the derivative of λ^2 has a horizontal tangent (Fig. 2). It would be convenient to cut the potential at this radius.

3.2 Solution of the radial Schroedinger equation

After having determined the potential function κ^2 which internally depends on E , we can solve the radial Schrödinger equation derived from (33):

$$-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} R(r) - \frac{\hbar^2}{mr} \frac{d}{dr} R(r) + V(r) R(r) = E R(r) \quad (42)$$

with R being the radial part of the wave function. We substitute R as usual:

$$P(r) := r R(r) \quad (43)$$

to obtain the simplified equation

$$\frac{d^2}{dr^2} P(r) = \frac{2m}{\hbar^2} (V(r) - E) P(r). \quad (44)$$

$V - E$ can be replaced by κ^2 which is already known from the constraint equation, so we have

$$\frac{d^2}{dr^2} P(r) = \frac{\lambda^2 P(r)}{r} = \kappa^2 P(r). \quad (45)$$

Obviously the energy parameter E is subsumed by κ . The computed κ function is valid for an arbitrary E . Since the left hand side of (45) is a replacement of the ∇^2 operator, the Schroedinger equation has been transformed into a Beltrami equation with variable scalar function κ^2 (assuming no divergence of P). There is no energy dependence left and the equation can be solved as an ordinary differential equation. This is a linear equation in P so that the result can be normalized arbitrarily and so can the final result R . This is the same again as for the solution procedure of the Schroedinger equation. Regarding the initial conditions, P starts at zero as discussed above and its derivative can be chosen arbitrarily, for example:

$$P(0) = 0, \quad (46)$$

$$\frac{dP}{dr}(0) = 1. \quad (47)$$

The results for R , R^2 and $R^2 r^2$ are graphed in Fig. 3. Again the functions have to be cut at the cut-off radius of about $2 \cdot 10^{-5}$ a.u.

3.3 Comparison with experiments

Experimental values of particle radii are listed in Table 1. The classical electron radius is calculated from equating the mass energy with the electrostatic energy in a sphere and turns out to be simply

$$r_e = \alpha^2 a_0 \quad (48)$$

with α being the fine structure constant and a_0 the Bohr radius. This radius value is however larger than the proton radius. Therefore a more realistic calculational procedure seems to be scaling the proton radius with the mass ratio compared to the electron (second row in Table 1). The experimental limits are even smaller so that the accepted opinion is that the electron is a point particle which it certainly cannot be in a mathematical sense since there are no singularities in nature.

The charge density characteristics of proton and neutron are exponentially decreasing functions. This is not totally identical to the properties obtained for R^2 from our calculation (Fig. 4) which more looks like a Gaussian function. However, Gaussians have been observed for atomic nuclei containing more than one proton and neutron.

Particle	charge density characteristic	radius [m]	radius [a.u.]
electron (classical)	delta function	$2.82 \cdot 10^{-15}$	$5.33 \cdot 10^{-5}$
electron (derived) ^a	delta function	$9.1 \cdot 10^{-17}$	$1.72 \cdot 10^{-6}$
proton (measured)	neg. exponential function	$1.11 \cdot 10^{-15}$	$2.10 \cdot 10^{-5}$
proton (charge radius)	neg. exponential function	$8.8 \cdot 10^{-16}$	$1.66 \cdot 10^{-5}$
neutron (measured)	neg. exponential function	$1.7 \cdot 10^{-15}$	$3.21 \cdot 10^{-5}$
atomic nuclei	Gaussian or Fermi function	$2 - 8 \cdot 10^{-15}$	$4 - 15 \cdot 10^{-5}$

^aElectron radius from volume comparison with $(m_{proton}/m_{electron})^{1/3}$

Table 1: Experimental data of elementary particles [1].

There is a diagram in the literature showing the charge densities for the proton and neutron [1] (reproduced in Fig. 5). The charge densities start with zero values therefore they seem to describe the effective charge in a sphere of radius r which has to be compared with

$$\rho_e = R^2 \cdot r^2 \quad (49)$$

of our calculation. This function (with negative sign) has been graphed in Fig. 4 in the range below the cut-off radius. Since our function is not normalized the vertical scales differ. The proton has a shoulder in the charge density which is not reproduced by our calculation. The neutron is known not to be charge-neutral over the radius but to have a positive core and a negative outer region. The negative region which is called "shell" even pertains to the centre in Fig. 5. The shape of the shell is quite conforming to our calculation in Fig. 4. Some other experimental charge densities of the proton have been derived by Venkat et al. [2], see Fig. 4 therein. They compare quite well with our results for $R^2 r^2$, Fig. 4 of this paper.

As already stated, our calculation does not contain an explicit energy parameter, therefore we do not obtain a mass spectrum of elementary particles or partons. The diameter of effective charge is defined by the initial value of κ^2 . For the results shown we had to choose $\kappa^2 = -5 \cdot 10^{10}$ a.u. which is quite a lot. The rest energy of the proton is 938 MeV or $3.5 \cdot 10^7$ a.u. which is three orders of magnitude less. Obviously the potential has to be much deeper than the (negative) rest energy.

In conclusion, the Beltrami approach of ECE theory leads to a qualitatively correct description of the internal structure of elementary particles, in particular the neutron. The binding energy cannot be determined since it cancels out from the calculation. It seems that the Beltrami structure is not valid in the boundary region of elementary particles or partons since the charge density does not go asymptotically to zero. This can be remedied by defining a cut-off radius where the radial function has a zero crossing. This was a first approach to compute the interior of elementary particles (the so-called parton structure) by ECE theory. For future developments more sophisticated approaches have to be found.

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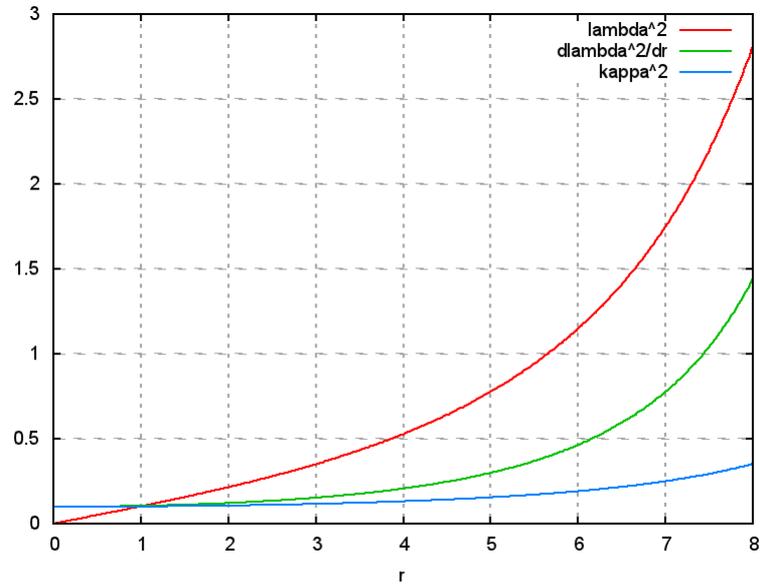


Figure 1: Solution functions of constraint equation (38) for $\kappa^2(0) > 0$.

<http://arxiv.org/pdf/1010.3629v2.pdf>

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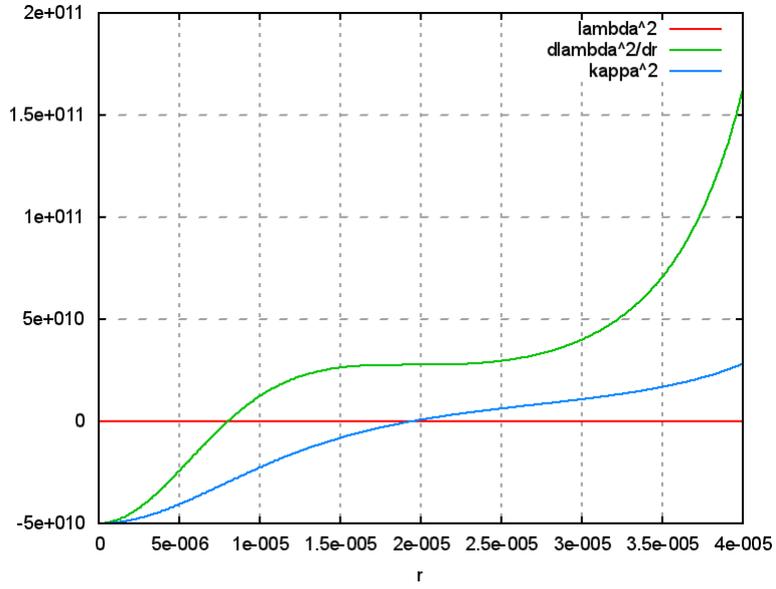


Figure 2: Solution functions of constraint equation (38) for $\kappa^2(0) < 0$.

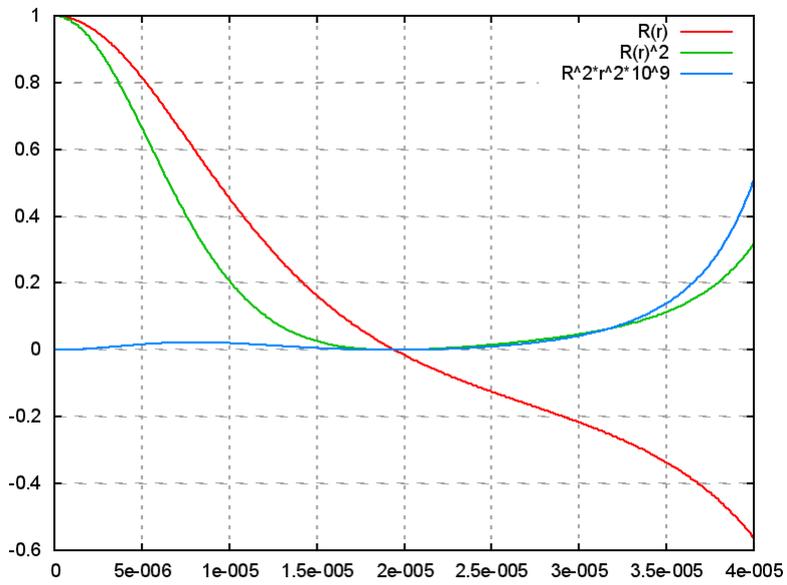


Figure 3: Parton solution of the Schroedinger equation.

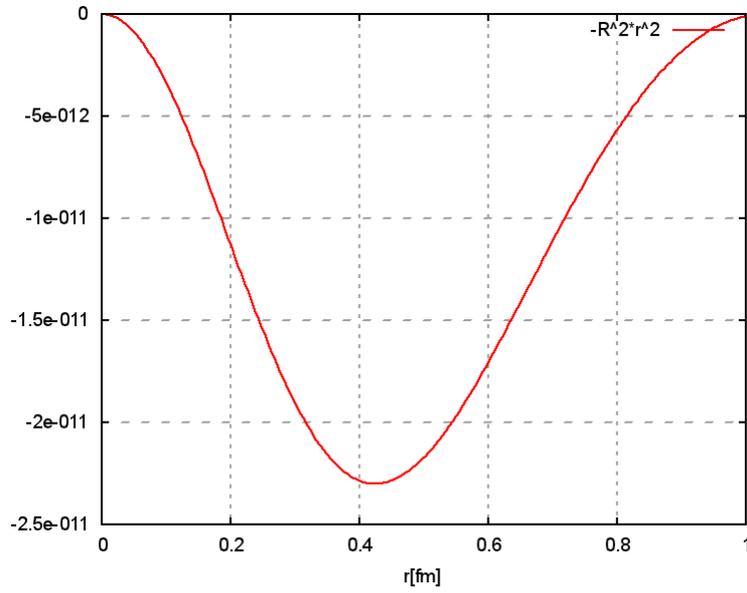


Figure 4: Radial wave function $-R^2 \cdot r^2$.

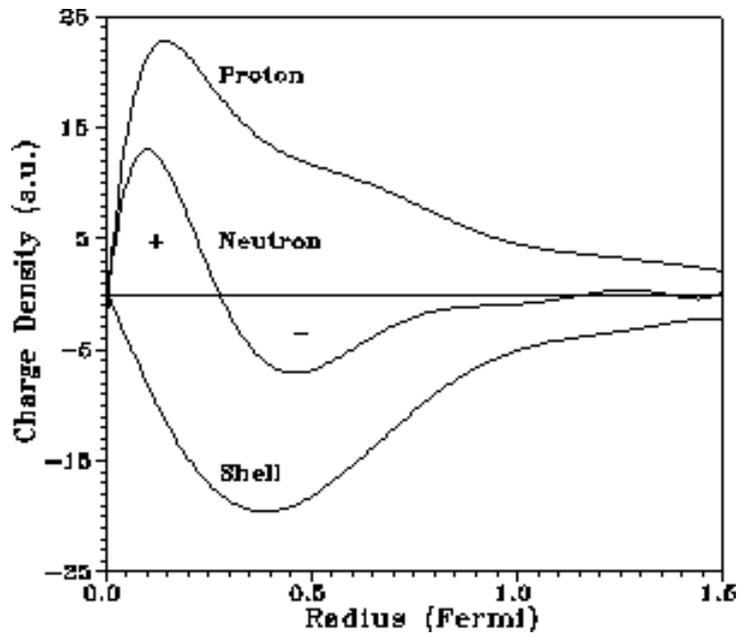


Figure 5: Experimental charge densities of elementary particles [1].