

THOMAS AND DE SITTER PRECESSION IN TERMS OF THE EVANS ECKART  
THEOREM OF ECE2 THEORY.

by

M. W. Evans and H. Eckardt

Civil List, AIAS and UPITEC

([www.archive.org](http://www.archive.org), [www.webarchive.org.uk](http://www.webarchive.org.uk), [www.aias.us](http://www.aias.us), [www.upitec.org](http://www.upitec.org),  
[www.atomicprecision.com](http://www.atomicprecision.com), [www.et3m.net](http://www.et3m.net) )

ABSTRACT

It is shown that the Evans / Eckardt Theorem of UFT342 can be used to describe Thomas and de Sitter precession to contemporary experimental precision by use of ECE2 relativity and the foundational definition of the relativistic momentum.

Keywords: ECE2 relativity, Evans / Eckardt Theorem, Thomas and de Sitter precession.

UFT 343

---

## 1. INTRODUCTION

In immediately preceding papers of this series {1 - 12} it has been shown that ECE2 relativity unifies special and general relativity and produces many new results, notably light deflection due to gravitation (UFT324 and UFT328) and orbital precession (UFT342) by consideration of the foundational definition of the relativistic momentum. To many scholars, this is considered to be the most fundamental definition of relativity, and is necessitated by conservation of momentum. In this paper the Evans Eckardt Theorem inferred in UFT324 is extended to give an exact description of Thomas and de Sitter precession to contemporary experimental precision. In the standard model, the Thomas precession is the rotation of the Minkowski infinitesimal line element, and the de Sitter precession is the rotation of the “Schwarzschild” line element. The Thomas precession is still valid, but the claimed derivation of the de Sitter precession is well known to be riddled with errors {1 - 12} because it is based on a geometry without torsion. In Section 2 it is shown that de Sitter precession can be derived correctly from ECE2 relativity to state of art experimental precision.

This paper is a brief synopsis of detailed calculations reported in its accompanying background notes posted with UFT343 on [www.aias.us](http://www.aias.us). Note 343(1) defines the Thomas precession and a lagrangian method is used to define the conserved angular momentum. The Thomas precession and phase shift are defined. Note 343(2) considers the Thomas precession in the Newtonian limit and derives a rotating conic section defined in rotating plane polar coordinates. The rotation takes place at a constant angular velocity. Note 343(3) derives the orbit of the de Sitter precession (the geodesic precession) using the observed precession of orbits in the static plane polar coordinate frame. The rotation of the frame of the precessing orbit is the de Sitter precession, or geodesic precession. Note 343(3) defines the Evans Eckardt Theorem needed for the description of de Sitter precession to state

of art experimental accuracy. Note 343(4) gives details of the calculation of the relativistic angular velocity produced by Thomas precession and gives details of the calculation of the precessing orbit.

## 2. THE ORBITS PRODUCED BY THOMAS AND DE SITTER PRECESSION.

Consider the Thomas frame rotation in the Newtonian limit:

$$\theta_1 = \theta + \omega_\theta t \quad - (1)$$

where  $\omega_\theta$  is the constant angular velocity of the frame rotation. The angle  $\theta_1$  is that of a rotating plane polar coordinate system defined by ( $r, \theta_1$ ). The total angular velocity is defined by:

$$\omega_1 = \frac{d\theta_1}{dt} = \frac{d\theta}{dt} + \omega_\theta. \quad - (2)$$

The lagrangian associated with the rotating frame in the Newtonian limit is:

$$\mathcal{L}_1 = \frac{1}{2} m v_1^2 - U \quad - (3)$$

where:

$$v_1^2 = \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta_1}{dt} \right)^2. \quad - (4)$$

The Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}_1}{\partial \theta_1} = \frac{d}{dt} \frac{\partial \mathcal{L}_1}{\partial \dot{\theta}_1}, \quad - (5)$$

$$\frac{\partial \mathcal{L}_1}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}_1}{\partial \dot{r}}, \quad - (6)$$

from which the conserved angular momentum in the rotating frame is:

$$L_1 = mr^2 \frac{d\theta}{dt} \quad - (7)$$

$$= L + \omega_0 mr$$

where

$$L = mr^2 \frac{d\theta}{dt} \quad - (8)$$

is the conserved angular momentum in the static frame ( $r, \theta$ ). Both  $L_1$  and  $L$  are constants of motion.

The hamiltonian in the rotating frame is:

$$H_1 = \frac{1}{2} m \left( \frac{dr}{dt} \right)^2 + \frac{1}{2} \frac{L_1^2}{mr^2} + U(r) \quad - (9)$$

where

$$U(r) = -\frac{mM G}{r} \quad - (10)$$

is the gravitational potential between a mass  $m$  orbiting a mass  $M$  at a distance  $r$ . Here  $G$  is Newton's constant. As shown in Note 343(2) the hamiltonian, a constant of motion in the rotating frame, produces the rotating conic section:

$$r = \frac{d_1}{1 + \epsilon_1 \cos(\theta + \omega_0 t)} \quad - (11)$$

As shown in Note 343(4):

$$t = \int \left( \frac{2}{m} (H - U) - \frac{L_1^2}{mr^2} \right)^{-1/2} \frac{dr}{\omega_0} \quad - (12)$$

Eqs. (11) and (12) can be solved simultaneously with computer algebra to give the orbit  $r$  in terms of  $\theta$ . As shown in note 343(4):

$$\theta = \cos^{-1} \left( \frac{1}{\epsilon_1} \left( \frac{d_1}{r} - 1 \right) \right) - \omega_\theta / \left( \frac{2}{m} (H-U) - \frac{L_1^2}{m^2 r^2} \right)^{-1/2} dr - (13)$$

and  $\theta$  can be plotted against  $r$ . Eq. (13) can be inverted numerically to give a plot of  $r$  against  $\theta$ . The Newtonian results are the well known:

$$\theta = \cos^{-1} \left( \frac{1}{\epsilon} \left( \frac{d}{r} - 1 \right) \right) - (14)$$

and

$$r = \frac{d}{1 + \epsilon \cos \theta}. - (15)$$

The orbit of de Sitter precession follows immediately as :

$$r = \frac{d_1}{1 + \epsilon_1 \cos(x\theta_1)} - (16)$$

where it is known experimentally that:

$$x = 1 - \frac{3M_G}{c^2 d_1} - (17)$$

in which the half right latitude of the rotating frame is:

$$d_1 = \frac{L_1^2}{m^2 M G} - (18)$$

and in which the eccentricity in the rotating frame is defined by:

$$\epsilon_1 = \left( 1 + \frac{2H_1 L_1}{m^3 M^2 G^2} \right). - (19)$$

The reason for Eq. (16) is that de Sitter or geodetic precession is defined by rotating the plane polar coordinate system in which the precession of a planar orbit is observed. The original method used by de Sitter was based on the then new Einstein field equation of 1915.

This equation is now well known and accepted to be incorrect due to neglect of torsion. In contrast, Eq. (16) is rigorously correct and based on ECE2 relativity, Lorentz covariant relativity in a space with non zero torsion and curvature.

The orbital velocity from Eq. (11) is:

$$v_{N1}^2 = \frac{L_1^2}{m^2 r^4} \left( r^2 + \left( \frac{dr}{d\theta_1} \right)^2 \right) - (20)$$

from which the relativistic velocity can be defined as in UFT342:

$$v^2 = v_{N1}^2 \left( 1 - \frac{v_{N1}^2}{c^2} \right)^{-1} - (21)$$

The relativistic velocity is the velocity given by Eq. (16):

$$v^2 = \frac{L^2}{m^2} \left( \frac{1}{r^2} + \frac{x^2 \epsilon^2}{d^2} \sin^2(\alpha\theta_1) \right) - (22)$$

so the Evans Eckardt Theorem for de Sitter precession to state of the art experimental

precision is:

$$\frac{L^2}{m^2} \left( \frac{1}{r^2} + \frac{x^2 \epsilon^2}{d^2} \sin^2(\alpha\theta_1) \right) = \frac{L_1^2 \left( \frac{1}{r^2} + \frac{\epsilon_1^2}{d_1^2} \sin^2\theta_1 \right)}{1 - \left( \frac{L_1}{mc} \right)^2 \left( \frac{1}{r^2} + \frac{\epsilon_1^2}{d_1^2} \sin^2\theta_1 \right)} - (23)$$

Eq. (23) can be developed with the methods of UFT342. Note carefully that both  $L$  and  $L_1$  are constants of motion. At the end of the calculation,  $\theta_1$  can be expressed as:

$$\theta_1 = \theta + \omega_0 t - (24)$$

Finally the velocity of the Thomas precession is the relativistic velocity:

$$\frac{v^2}{r} = \frac{L_1^2 \left( \frac{1}{r^2} + \frac{\epsilon_1^2}{d_1^2} \sin^2 \theta_1 \right)}{1 - \left( \frac{L_1}{mr} \right)^2 \left( \frac{1}{r^2} + \frac{\epsilon_1^2}{d_1^2} \sin^2 \theta_1 \right)} \quad -(25)$$

and the Thomas angular velocity (the relativistic angular velocity) is:

$$\Omega_T = v_T / r. \quad -(26)$$

This is used as in UFT110 to define the Thomas phase shift. The latter can be observed in a Foucault pendulum as is well known.

#### ACKNOWLEDGMENTS

The British Government is thanked for the award of a Civil List Pension, and the staff of AIAS and others are thanked for many interesting discussions. Dave Burleigh is thanked for site maintenance, feedback software maintenance and posting, Alex Hill for translation and broadcasting, and Robert Cheshire for broadcasting.

# Thomas and de Sitter precession in terms of the Evans Eckardt Theorem of ECE2 theory

M. W. Evans\*, H. Eckardt†  
Civil List, A.I.A.S. and UPITEC

([www.webarchive.org.uk](http://www.webarchive.org.uk), [www.aias.us](http://www.aias.us),  
[www.atomicprecision.com](http://www.atomicprecision.com), [www.upitec.org](http://www.upitec.org))

## 3 Numerical analysis of orbits

The orbits for Thomas and de Sitter precession will be analysed. The equations (11) and (12) have to be solved simultaneously for  $r$  and  $t$ . Eq.(13) can be used to obtain  $\theta$  if the orbit  $r$  is known. For a Newtonian frame  $(r, \theta_1)$  in which the ellipse is stationary, it is

$$\theta_1 = \theta + \omega_\theta t \quad (27)$$

and the radius function is

$$r = \frac{\alpha_1}{1 + \epsilon_1 \cos(x\theta_1)} \quad (28)$$

where we have Thomas precession for  $x = 1$  and de Sitter precession (with additional rotation of the elliptic axes) for  $x \neq 1$ . Computation of the time dependence of  $\theta$  could be done by solving the integral in (12) either analytically or numerically, but we use a simpler method derived in UFT paper 238, Eq.(148/203):

$$t = \frac{2\alpha_1^2 m}{x L_1} \left( \frac{\operatorname{atan} \left( \frac{(2\epsilon_1 - 2) \sin(\theta_1 x)}{2\sqrt{1-\epsilon_1^2} (\cos(\theta_1 x) + 1)} \right)}{\sqrt{1 - \epsilon_1^2} (\epsilon_1^2 - 1)} - \frac{\epsilon_1 \sin(\theta_1 x)}{(\cos(\theta_1 x) + 1) \left( \frac{(\epsilon_1^3 - \epsilon_1^2 - \epsilon_1 + 1) \sin(\theta_1 x)^2}{(\cos(\theta_1 x) + 1)^2} - \epsilon_1^3 - \epsilon_1^2 + \epsilon_1 + 1 \right)} \right). \quad (29)$$

We want to show how the ellipse rotates in a fixed frame with coordinates  $r$ ,  $\theta$  and  $t$ . The time  $t$  relates to the motion in the Newtonian frame as well as to the rotating frame. A complication is introduced by the fact that via (27) the angle  $\theta_1$  depends additionally on time, when considered from the fixed lab frame. Consequently,  $\theta_1$  is not an independent variable. An iterative solution

---

\*email: emyrone@aol.com

†email: mail@horst-eckardt.de

procedure has been designed as follows. We define a grid of one-dimensional angular values  $\theta_n$  etc. and compute the sequence

$$\theta_n = \theta_{n-1} + \Delta\theta \quad (30)$$

$$\theta_{1,n} = \theta_n + \omega_\theta t_{n-1} \quad (31)$$

$$t_n = t(\theta_{1,n}) \quad (32)$$

$$r_n = r(\theta_{1,n}) \quad (33)$$

with a fixed increment  $\Delta\theta$ . This leads to a numerical evaluation of the functions  $r(\theta)$  and  $t(\theta)$  which are graphed in Fig. 1 with numerical parameters  $G = M = m = \alpha_1 = 1$ ,  $L_1 = 5$ ,  $H = -0.5$ ,  $\epsilon_1 = 0.3$ . We first study the effect of  $\omega_\theta$ . For a static ellipse we have  $\omega_\theta = 0$ . The time function as well as the radius function are scaled horizontally when switching to  $\omega_\theta = 0.5$ . The radius function is graphed in Fig. 2 as a polar diagram for both  $\omega_\theta$  values. There is a clear precession if  $\omega_\theta > 0$ . The reverse precession occurs if  $\omega_\theta < 0$  (not shown). This is an example for orbital or Thomas precession. A de Sitter precession can be added by setting  $x \neq 0$ , for example  $x = 0.95$  as done for Fig. 3. Now the original ellipse (for  $\omega_\theta = 0$ ) precesses. When orbital precession is added (by  $\omega_\theta > 0$ , see Fig. 3), the orbital precession is compensated in part by the de Sitter precession. Both types of precession can give an increase or decrease of total precession, depending on the sign of  $\omega_\theta$  and the condition  $x > 1$  or  $x < 1$ .

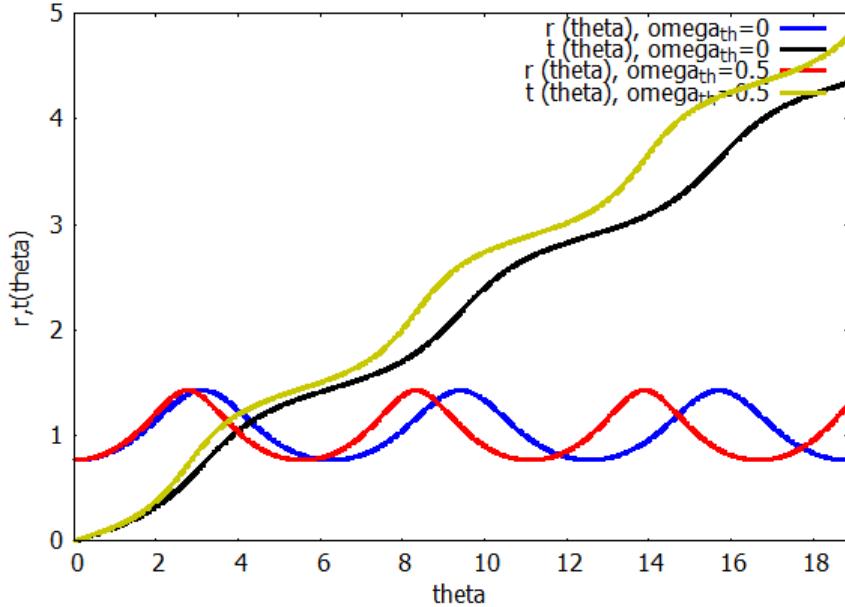


Figure 1: Orbit  $r(\theta)$  and time  $t(\theta)$  for a static ellipse ( $\omega_\theta = 0$ ) and Thomas precession ( $\omega_\theta > 0$ ).

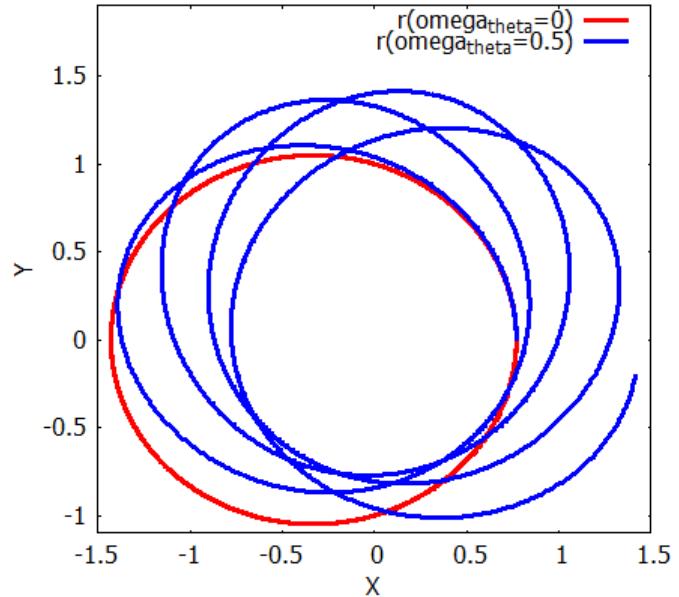


Figure 2: Polar plot of orbit  $r(\theta)$  for a static ellipse (red) and Thomas precession (blue).

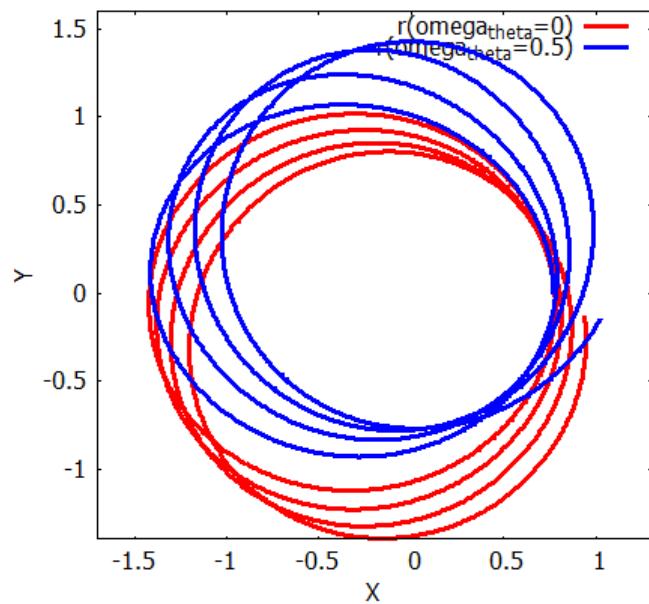


Figure 3: Polar plot of orbit  $r(\theta)$ ,  $x = 0.95$ , for de Sitter precession (red) and de Sitter plus Thomas precession (blue).

## REFERENCES

- {1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE Theory" (UFT281 - UFT288 and Spanish Section, New Generation Publishing in prep.).
- {2} M .W. Evans, Ed., J. Found. Phys. Chem., (Cambridge International Science Publishing, CISP, 2011, and relevant UFT papers).
- {3} M .W. Evans, Ed., "Definitive Refutations of Einsteinian General Relativity" (special issue of ref. (2)).
- {4} M .W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 and CISP 2010).
- {5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007 and UFT302, translated by Alex Hill, Spanish language section of [www.aias.us](http://www.aias.us)).
- {6} H. Eckardt, "The ECE Engineering Model" (UFT303).
- {7} M. W. Evans, "Book of Scientometrics" (UFT307).
- {8} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis 2005 to 2011 in seven volumes, and relevant UFT papers).
- {9} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001 and Omnia Opera of [www.aias.us](http://www.aias.us)).
- {10} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.
- {11} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer 1994 to 2002 in five volumes each hardback and softback and Omnia Opera of [www.aias.us](http://www.aias.us))
- {12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).