

# PRECESSION FROM THE ECE2 COVARIANT MINKOWSKI FORCE EQUATION.

by

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## ABSTRACT

It is shown that the ECE2 covariant Minkowski force equation produces orbital precession in a plane. The equation is derived with ECE2 covariant Euler Lagrange equations which are inferred for the first time in this paper. These equations show the existence of an ECE2 covariant Hamilton Principle of Least Action. The precession from the relativistic Euler Lagrange equations is compared with precession derived from ECE2 fluid gravitation.

Keywords: ECE2 covariant Minkowski force equation, ECE2 covariant Euler Lagrange equation, comparison of precession theories.

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## 1. INTRODUCTION

In recent papers of this series {1 - 12}, ECE2 covariant theories of orbital precession have been developed using numerical integration of simultaneous differential equations. In Section 2 the ECE2 covariant Euler Lagrange equations are inferred for the first time, these equations contain the proper time  $\tau$  and derive ultimately from an ECE2 covariant Hamilton Principle of Least Action. These equations are used to infer the ECE2 covariant Minkowski force equation of a planar orbit, an equation which produces orbital precession. This type of precession theory is compared with precession from ECE2 covariant fluid dynamics.

This paper is a brief synopsis of extensive calculations in the notes accompanying UFT376 on [www.aias.us](http://www.aias.us). In Note 376(1) the theory is given of precession from fluid gravitation, and the complete set of ECE2 gravitational field equations written out. Note 376(2) gives the equations of gravitostatics and gravitomagnetostatics. Notes 376(3) and 376(6) give the ECE2 covariant Euler Lagrange equations, and compare them with the equations of precession due to fluid spacetime. Finally Note 376(7) defines the relativistic angular momentum as a constant of motion, using the derivative with respect to the proper time.

## 2. THE ECE2 COVARIANT EULER LAGRANGE EQUATIONS.

Consider the ECE2 covariant lagrangian of recent UFT papers {1 - 12}:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad (1)$$

where  $\gamma$  is the Lorentz factor,  $m$  is a mass orbiting a mass  $M$  in two or three dimensions,  $c$  is the vacuum speed of light and  $U$  is the potential energy. Denote  $\underline{r}$  as the

vector distance between m and M. The Lorentz factor is then defined as:

$$\gamma = \left( 1 - \frac{\dot{\underline{r}} \cdot \dot{\underline{r}}}{c^2} \right)^{-1/2} \quad - (2)$$

The relativistic momentum is defined by:

$$\underline{p} = \gamma m \underline{v} = \gamma m \dot{\underline{r}} = \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} = m \frac{d\underline{r}}{d\tau} = \gamma m \frac{d\underline{r}}{dt} \quad - (3)$$

where  $\tau$  is the proper time, and the relativistic or Minkowski force is the derivative of the relativistic momentum with respect to proper time:

$$\underline{F} = \frac{d\underline{p}}{d\tau} = \gamma \frac{d(\gamma \underline{p}_0)}{dt} = \gamma^4 m \frac{d^2 \underline{r}}{dt^2} \quad - (4)$$

In orbital motion this force is also defined as:

$$\underline{F} = - \partial U / \partial \underline{r} \quad - (5)$$

where  $U(\underline{r})$  is the potential energy of attraction between m and M. Therefore the ECE2 covariant orbital equation is:

$$\underline{F} = \gamma^4 m \frac{d^2 \underline{r}}{dt^2} = -mM\gamma \frac{\underline{r}}{r^3} \quad - (6)$$

For a central potential of type:

$$U = -\frac{mM\gamma}{r} \quad - (7)$$

Eq. (6) is given by the ECE2 covariant Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad - (8)$$

Q. E. D. Eq. (8) is inferred for the first time in this paper and implies the existence of an ECE2 covariant Hamilton Principle of Least Action. Note carefully that the proper time

$\tau$  must be used on the right hand side of Eq. ( 8 ).

Therefore the ECE2 covariant orbital equation is:

$$\gamma^4 \ddot{\underline{r}} = -mG \frac{\underline{r}}{r^3} \quad - (9)$$

This gives a precessing orbit as shown with numerical methods in Section 3.

The covariant orbital equation must be solved with the gravitational field equations of ECE2, given in previous UFT papers and written out in Note 376(1). If the

gravitomagnetic field  $\underline{\Omega}$  is neglected the field equations become those of gravitostatics:

$$\begin{aligned} \underline{\nabla} \times \underline{g} &= \underline{0} \quad - (10) \\ \underline{\nabla} \cdot \underline{g} &= \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad - (11) \\ \frac{d\underline{g}}{dt} &= \underline{0} \quad - (12) \end{aligned}$$

where the vector  $\underline{\kappa}$  is related to the spin connection vector. Here G is Newton's constant and  $\rho_m$  is the mass density. Therefore the orbit must be found by solving Eq. ( 9 ) numerically and using the field equations ( 10 ) to ( 12 ) as constraints. The Minkowski equation is the ECE2 covariant Newton equation.

As shown in immediately preceding UFT papers the force equation of ECE2 fluid gravitation is:

$$\underline{F} = m\underline{g} = m \left( \frac{d\underline{v}}{dt} + (\underline{v} \cdot \underline{\nabla}) \underline{v} \right) = -mM G \frac{\underline{r}}{r^3} \quad - (13)$$

in which appears the convective derivative of the velocity field of spacetime:

$$\underline{v} = \underline{v} (\dot{X}(t), \dot{Y}(t), t) \quad - (14)$$

The velocity field depends on X(t), Y(t) and t, so is a function of a function. The velocity field is the convective derivative of the position element  $\underline{R}(X(t), Y(t), t)$  as follows:

$$\underline{v} = \frac{\partial \underline{R}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{R}. \quad - (15)$$

The two theories give the same orbital precession if:

$$\gamma^4 \frac{d^2 \underline{r}}{dt^2} = \frac{d\underline{v}}{dt} + (\underline{v} \cdot \underline{\nabla}) \underline{v}. \quad - (16)$$

In the Newtonian limit:

$$\underline{v}(\dot{x}(t), \dot{y}(t), t) \rightarrow \underline{v}(t) \quad - (17)$$

and

$$\gamma \rightarrow 1. \quad - (18)$$

It can be inferred from Eq. ( 16 ) that the effect of a fluid spacetime is to change the Newton equation into the Minkowski equation. Therefore fluid spacetime induces orbital precession. In the Newtonian limit Eq. ( 16 ) reduces to:

$$\underline{v} = \dot{\underline{r}}. \quad - (19)$$

The equations of fluid spacetime are the Kambe equations {1 - 12}:

$$\underline{\nabla} \cdot \underline{E}_F = \rho_F \quad - (20)$$

$$\underline{\nabla} \cdot \underline{B}_F = 0 \quad - (21)$$

$$\underline{\nabla} \times \underline{E}_F + \frac{\partial \underline{B}_F}{\partial t} = \underline{0} \quad - (22)$$

$$\underline{\nabla} \times \underline{B}_F - \frac{1}{a_0^2} \frac{\partial \underline{E}_F}{\partial t} = \frac{1}{a_0^2} \underline{J}_F \quad - (23)$$

where E is the Kambe electric field:

$$\underline{E}_F = (\underline{v}_F \cdot \underline{\nabla}) \underline{v}_F = -\underline{\nabla} \Phi_F - \frac{\partial \underline{v}_F}{\partial t} \quad - (24)$$

where  $\Phi_F$  is a potential term. The Kambe magnetic field is the vorticity of the fluid spacetime:

$$\underline{B}_F = \underline{\nabla} \times \underline{v}_F. \quad (25)$$

The quantity  $a_0$  is the speed of sound and  $\underline{J}_F$  is the Kambe current {1 - 12}. The set of equations (20-23) has the same structure as the ECE2 covariant gravitational field equations:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad (26)$$

$$\underline{\nabla} \times \underline{g} + \partial \underline{\Omega} / \partial t = \underline{0} \quad (27)$$

$$\underline{\nabla} \cdot \underline{g} = \kappa \cdot \underline{g} = 4\pi G \rho_m \quad (28)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \partial \underline{g} / \partial t = \kappa \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_M \quad (29)$$

$$\rho_m / \partial t + \underline{\nabla} \cdot \underline{J}_M = 0 \quad (30)$$

where  $\underline{\Omega}$  is the gravitomagnetic field,  $\rho_m$  the mass density and  $\underline{J}_M$  the mass current density.

If the vorticity is neglected, the Kambe equations reduce to:

$$\underline{\nabla} \cdot \underline{E}_F = \nabla_F \quad (31)$$

$$\underline{\nabla} \times \underline{E}_F = \underline{0} \quad (32)$$

$$\partial \underline{E}_F / \partial t = \underline{0} \quad (33)$$

so

$$\underline{g} = \frac{\partial \underline{v}_F}{\partial t} + \underline{E}_F. \quad (34)$$

The lagrangian (1) can be developed in plane polar coordinates (r,  $\phi$ ) if

$$\underline{v}^2 = \underline{\dot{r}} \cdot \underline{\dot{r}} = \dot{r}^2 + r^2 \dot{\phi}^2 \quad (35)$$

The proper Lagrange variables are  $r$  and  $\phi$ , and the Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{d\tau} \left( \frac{\partial \mathcal{L}}{\partial \dot{r}} \right) - (36)$$

and

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{d\tau} \left( \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - (37)$$

These are ECE2 covariant, so the proper time appears on the right hand side. In Eq. ( 36 ):

$$p = m \frac{\partial \mathcal{L}}{\partial \dot{r}} = m \gamma \dot{r} - (38)$$

and in Eq. ( 37 ):

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \gamma r^2 \dot{\phi} - (39)$$

It follows that:

$$F = m \frac{dp}{d\tau} = -\frac{mM\phi}{r^2} - (40)$$

The relativistic angular momentum is:

$$L = \gamma m r^2 \dot{\phi} = \gamma m r^2 \frac{d\phi}{dt} - (41)$$

and from Eq. ( 37 ) it is conserved as follows:

$$\frac{dL}{d\tau} = \gamma \frac{dL}{dt} = 0 - (42)$$

The ECE2 covariant torque is:

$$\overline{T}_\tau = \frac{dL}{d\tau} - (43)$$

and the relativistic angular momentum is:

$$L = \gamma L_0 \quad - (44)$$

where:

$$L_0 = m r^2 \frac{d\phi}{dt}, \quad - (45)$$

the non relativistic angular momentum is:

$$L_0 = \frac{L}{\gamma} \quad - (46)$$

Therefore:

$$\frac{dL}{d\tau} = \frac{dL}{dt} = \gamma \frac{dL}{dt} = \gamma \frac{d(\gamma L_0)}{dt} = 0 \quad - (47)$$

The relativistic angular momentum does not change with proper time:

$$\frac{dL}{d\tau} = 0 \quad - (48)$$

i. e.

$$\gamma \frac{dL}{dt} = 0 \quad - (49)$$

From Eq. (47):

$$\frac{dL}{dt} = L_0 \frac{d\gamma}{dt} + \gamma \frac{dL_0}{dt} = 0 \quad - (50)$$

so in general:

$$\frac{dL_0}{dt} \neq 0, \quad - (51)$$

i.e. the non relativistic angular momentum is not conserved in a relativistic theory.

### 3. NUMERICAL DEMONSTRATION OF PRECESSION

Section by Dr. Horst Eckardt

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