COMPLETE SOLUTIONS OF THE ECE2 FIELD EQUATIONS.

by

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ABSTRACT

Complete solutions of the ECE2 field equations are given for an electromagnetic and gravitational free space plane wave, a static magnetic flux density, **B**, a static gravitomagnetic field, a static electric field strength **E** and the gravitostatic acceleration due to gravity **g**. In each case the complete solutions include the spin connection four vector components.

Keywords, ECE2, complete solutions for the free space plane wave and static fields.

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INTRODUCTION

In the immediately preceding paper (UFT380) work was initiated towards a complete solution of the ECE2 covariant field equations {1 - 12} of electromagnetism and gravitation. These complete solutions are also solutions of the ECE2 hydrodynamic field equations. The general solution requires consideration of a set of seven non-linear partial differential equations in seven unknowns, the three components of the vector potential and the four components of the spin connection four vector. In Section 2, the solutions are written out in full for a free space plane wave of electromagnetism and gravitation, and for static fields in electromagnetism and gravitation.

This paper is a brief synopsis of detailed calculations contained in the notes accompanying UFT381 on www.aias.us and www.upitec.org (referred to as "combined sites"). Note 381(1) gives the complete solution for plane waves of electromagnetism and gravitation in free space, solutions which include the relevant spin connections and which obey the antisymmetry laws of ECE2. Note 381(2) gives particular solutions of the antisymmetry laws. Note 381(3) uses these particular solutions to give the complete solution for the static magnetic flux density ($\underline{B}$) and static gravitomagnetic field ($\underline{\Omega}$). Notes 381(4) to 381(8) gives the complete solution for the static electric field strength $\underline{E}$.

Section 3 gives a numerical and graphical analysis of selected solutions.

2. SOME COMPLETE SOLUTIONS

The ECE2 covariant field equations of electrodynamics {1 - 12} are:

$$\underline{\nabla} \cdot \underline{B} = \underline{\kappa} \cdot \underline{B} \quad -(1)$$

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} = \rho / \epsilon_0 \quad -(2)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = -\left(\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E}\right) \quad -(3)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} - \frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B} = \mu_0 \underline{J} \quad -(4)$$

$$\kappa_0 = 2\left(\frac{q_0}{r^{(0)}} - \frac{\omega_0}{c}\right) = 2\left(\frac{A_0}{A^{(0)} r^{(0)}} - \frac{\omega_0}{c}\right) \quad -(5)$$

$$\underline{\kappa} = 2\left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right) = 2\left(\frac{\underline{A}}{A^{(0)} r^{(0)}} - \underline{\omega}\right) \quad -(6)$$

in the notation of UFT316 and UFT317. Here $\underline{B}$ is the magnetic flux density, $\underline{E}$ the electric field strength, ρ is the electric charge density, ϵ_0 is the vacuum permittivity in S. I. Units, $\underline{J}$ is the electric current density, and μ_0 is the vacuum permeability. The components of the kappa four vector:

$$\kappa^\mu = (\kappa_0, \underline{\kappa}) \quad -(7)$$

are defined by:

$$\kappa_0 = 2\left(\frac{q_0}{r^{(0)}} - \frac{\omega_0}{c}\right) \quad -(8)$$

and

$$\underline{\kappa} = 2\left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right) \quad -(9)$$

The spin connection four vector is defined by:

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega}\right) \quad -(10)$$

and the potential four vector is defined by:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right) \quad -(11)$$

where ϕ is the scalar potential and $\underline{A}$ is the vector potential. The relevant S. I. Units are as follows:

$$\begin{aligned} \underline{E} &= \text{volt}\, m^{-1} = J C^{-1} m^{-1} \\ \underline{A} &= J s\, C^{-1} m^{-1} \\ \phi &= \text{volt} = J C^{-1} \\ \underline{B} &= J s\, C^{-1} m^{-2} \\ \omega_0 &= s^{-1}, \quad \underline{\omega} = m^{-1} \end{aligned} \quad -(12)$$

The components of the kappa four vector are:

$$\kappa_0 = 2 \left(\frac{A_0}{A^{(0)} r^{(0)}} - \frac{\omega_0}{c} \right) \quad -(13)$$

and

$$\underline{\kappa} = 2 \left(\frac{\underline{A}}{A^{(0)} r^{(0)}} - \underline{\omega} \right) \quad -(14)$$

in which the ECE hypothesis has been used:

$$A_0 = A^{(0)} q_0 \quad -(15)$$

$$\underline{A} = A^{(0)} \underline{q} \quad -(16)$$

In general, the field equations allow for the existence of a magnetic charge / current density, i.e. a magnetic monopole (or charge density) and a magnetic current density. The magnetic charge density is zero if and only if:

$$\underline{\kappa} \cdot \underline{B} = 0 \quad -(17)$$

$$\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} = \underline{0}. \quad -(18)$$

In free space:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(19)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(20)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad -(21)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(22)$$

and

$$\underline{\kappa} \cdot \underline{E} = \rho / \epsilon_0 = 0 \quad -(23)$$

$$\frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B} = \mu_0 \underline{J} = \underline{0}. \quad -(24)$$

Eqs. (17) to (22) are satisfied by the plane waves:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(25)$$

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{i\phi} \quad -(26)$$

where:

$$\phi = \omega t - \kappa Z \quad -(27)$$

and where ω is the angular frequency of the wave at a point Z and instant t. The wave vector is defined by

$$\kappa = \frac{\omega}{c}. \quad -(28)$$

Eqs. (23) to (24) are satisfied by:

$$\kappa_0 = 0 \quad -(29)$$

$$\underline{\kappa} = \underline{0} \quad -(30)$$

so:

$$q_0 = r^{(0)} \frac{\omega_0}{c} \qquad -(31)$$

and

$$\underline{q} = r^{(0)} \underline{\omega} . \qquad -(32)$$

The electric field strength is given by:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \qquad -(33)$$

and the magnetic flux density by:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} . \qquad -(34)$$

When used with:

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad -(35)$$

Eqs. (33) and (34) imply:

$$\frac{\partial}{\partial t}\left(\underline{\omega} \times \underline{A}\right) + \underline{\nabla} \times \left(\omega_0 \underline{A}\right) = \underline{0} . \qquad -(36)$$

The antisymmetry laws from Eq. (34) imply:

$$\left(\frac{\partial}{\partial Y} - \omega_Y\right) A_Z = -\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_Y \qquad -(37)$$

$$\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_X = -\left(\frac{\partial}{\partial X} - \omega_X\right) A_Z \qquad -(38)$$

$$\left(\frac{\partial}{\partial X} - \omega_X\right) A_Y = -\left(\frac{\partial}{\partial Y} - \omega_Y\right) A_X \qquad -(39)$$

Considering the plane wave potential:

$$\underline{A} = \frac{A^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{i\phi} \quad -(40)$$

the antisymmetry laws reduce to:

$$\partial A_X / \partial Z = \omega_Z A_X = -i\kappa_Z A_X \quad -(41)$$

$$\partial A_Y / \partial Z = \omega_Z A_Y = -i\kappa_Z A_Y \quad -(42)$$

$$-\omega_X A_Y = \omega_Y A_X \quad -(43)$$

It follows that:

$$\mathrm{Real}\left(\omega_Z\right) = 0 \quad -(44)$$

and by inspection, the spin connection is the plane wave:

$$\underline{\omega} = \frac{\omega^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{-i\phi} \quad -(45)$$

From Eq. (32):

$$\underline{\omega} = \frac{\underline{A}^*}{A^{(0)} r^{(0)}} \quad -(46)$$

where $\underline{A}^*$ is the complex conjugate of $\underline{A}$:

$$\underline{A}^* = \frac{A^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{-i\phi} \quad -(47)$$

Finally, Eq. (36) implies that:

$$\omega_0 = 0 \quad -(48)$$

The complete solution for the free space plane waves in the absence of a magnetic charge / current desnity is therefore:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi}$$
$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{i\phi} \qquad -(49)$$
$$\underline{A} = \frac{A^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{i\phi}$$
$$\kappa_0 = 0, \quad \underline{\kappa} = \underline{0}$$
$$\omega_0 = 0, \quad \underline{\omega} = \frac{\omega^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{-i\phi}.$$

This is a simple example solution. In general, as described in UFT380, the homogeneous field equations:

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad -(50)$$
$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \qquad -(51)$$

together with Eq. (37) to Eq. (39) give seven equations in seven unknowns. These equations are given in UFT380.

The antisymmetry laws (37) to (39) are fundamental to physics, as discussed in UFT131 - UFT134 and in UFT350. They are a rigorous constraint and allow only certain types of solution. Note 381(2) discusses some particular solutions such as:

$$\frac{\partial A_Z}{\partial Y} = -\frac{\partial A_Y}{\partial Z}, \quad -\omega_Y A_Z = \omega_Z A_Y \qquad -(52)$$
$$\frac{\partial A_X}{\partial Z} = -\frac{\partial A_Z}{\partial X}, \quad -\omega_Z A_X = \omega_X A_Z \qquad -(53)$$

$$\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y}, \quad -\omega_X A_Y = \omega_Y A_X. \quad -(54)$$

Another set of particular solutions is:

$$\frac{\partial A_Z}{\partial Y} = \omega_Z A_Y \quad -(55)$$

$$\frac{\partial A_X}{\partial Z} = \omega_X A_Z \quad -(56)$$

$$\frac{\partial A_Y}{\partial X} = \omega_Y A_X. \quad -(57)$$

These particular solutions are useful for finding the static magnetic flux density from the ECE2 field equations as follows.

In two dimensions:

$$\left(\frac{\partial}{\partial X} - \omega_X\right) A_Y = -\left(\frac{\partial}{\partial Y} - \omega_Y\right) A_X \quad -(58)$$

which has self consistent particular solutions:

$$\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y}, \quad -(59)$$

$$-\omega_X A_Y = \omega_Y A_X. \quad -(60)$$

Consider the well known {1 - 12} magnetic planar vector potential:

$$\underline{A} = \frac{B^{(0)}}{2}\left(-Y\underline{i} + X\underline{j}\right) \quad -(61)$$

It follows that:

$$\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y} \quad -(62)$$

which is Eq. (59), Q. E. D. Using:

$$A_x = -\frac{B^{(0)}}{2} Y \,; \quad A_Y = \frac{B^{(0)}}{2} X \quad -(63)$$

Eq. (60) gives:

$$Y \omega_X = X \omega_Y . \quad -(64)$$

Furthermore:

$$\frac{\partial A_X}{\partial Y} = -\frac{B^{(0)}}{2} , \quad \frac{\partial A_Y}{\partial X} = \frac{B^{(0)}}{2} . \quad -(65)$$

From Eqs (57), (60), (64) and (65):

$$\omega_Y = -\frac{1}{Y} , \quad \omega_X = -\frac{1}{X} \quad -(66)$$

and the spin connection vector is:

$$\underline{\omega} = -\left(\frac{1}{X} \underline{i} + \frac{1}{Y} \underline{j} \right) \quad -(67)$$

Q. E. D. It follows as in Note 381(3) that:

$$\underline{\omega} \times \underline{A} = -B^{(0)} \underline{k} \quad -(68)$$

and:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} = 2B^{(0)} \underline{k} \quad -(69)$$

which is the required static magnetic flux density in the Z axis, Q. E. D.

The electric field strength is zero in magnetostatics, so:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} = \underline{0} . \quad -(70)$$

There is no time dependence in magnetostatics, so:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(71)$$

It follows that:

$$\omega_0 = 0 \quad -(72)$$

and that Eq. (36) reduces to:

$$\frac{\partial}{\partial t}\left(\underline{\omega} \times \underline{A}\right) = \underline{0}. \quad -(73)$$

This is true from Eq. (68), Q. E. D. :

$$\frac{\partial}{\partial t}\left(\underline{\omega} \times \underline{A}\right) = -\frac{\partial B^{(0)}}{\partial t}\underline{k} = \underline{0}. \quad -(74)$$

The ECE2 Ampère Law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(75)$$

means that $\underline{J}$ vanishes for the magnetic flux density (69). A net current density of zero is consistent with the fact that the electric charge density is zero because there is no electric field strength present.

The complete solution for the static magnetic flux density $\underline{B}$ is therefore:

$$\begin{aligned} \underline{B} &= 2B^{(0)}\underline{k}, \quad \underline{E} = \underline{0}, \\ \underline{A} &= \frac{B^{(0)}}{2}\left(-Y\underline{i} + X\underline{j}\right) \\ \omega_X &= -\frac{1}{X}, \quad \omega_Y = -\frac{1}{Y}. \end{aligned} \quad -(76)$$

The ECE2 field equations for the static electric field strength $\underline{E}$ are:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(77)$$

$$\underline{\nabla} \times \underline{E} = 0 \quad -(78)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(79)$$

$$\partial \underline{E} / \partial t = \underline{0} \quad -(80)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} = \underline{0}. \quad -(81)$$

Eq. (77) is the antisymmetry law of ECE2 electrostatics. Eq. (81) in component form gives three equations:

$$\frac{\partial A_Z}{\partial Y} - \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z - \omega_Z A_Y \quad -(82)$$

$$\frac{\partial A_X}{\partial Z} - \frac{\partial A_Z}{\partial X} = \omega_Z A_X - \omega_X A_Z \quad -(83)$$

$$\frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y} = \omega_X A_Y - \omega_Y A_X. \quad -(84)$$

Eqs. (77) and (78) give:

$$\frac{\partial}{\partial t} \underline{\nabla} \times \underline{A} + \underline{\nabla} \times (\omega_0 \underline{A}) = \underline{0}. \quad -(85)$$

As for magnetostatics:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(86)$$

so

$$\underline{\nabla} \times (\omega_0 \underline{A}) = \omega_0 \underline{\nabla} \times \underline{A} + \underline{A} \times \underline{\nabla} \omega_0 = \underline{0} \quad -(87)$$

which gives three components equations:

$$\omega_0 \left(\frac{\partial A_Z}{\partial Y} - \frac{\partial A_Y}{\partial Z} \right) + A_Z \frac{\partial \omega_0}{\partial Y} - A_Y \frac{\partial \omega_0}{\partial Z} = 0 \quad -(88)$$

$$\omega_0 \left(\frac{\partial A_X}{\partial Z} - \frac{\partial A_Z}{\partial X} \right) + A_X \frac{\partial \omega_0}{\partial Z} - A_Z \frac{\partial \omega_0}{\partial X} = 0 \quad -(89)$$

$$\omega_0 \left(\frac{\partial A_Y}{\partial Z} - \frac{\partial A_X}{\partial Y} \right) + A_Y \frac{\partial \omega_0}{\partial X} - A_X \frac{\partial \omega_0}{\partial Y} = 0 \quad -(90)$$

The Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(91)$$

gives:

$$\frac{\partial}{\partial t} \underline{\nabla} \cdot \underline{A} + \omega_0 \underline{\nabla} \cdot \underline{A} + \underline{A} \cdot \underline{\nabla} \omega_0 = -\rho / \epsilon_0 \quad -(92)$$

which in component format is:

$$\frac{\partial}{\partial t} \left(\frac{\partial A_X}{\partial X} + \frac{\partial A_Y}{\partial Y} + \frac{\partial A_Z}{\partial Z} \right) + \omega_0 \left(\frac{\partial A_X}{\partial X} + \frac{\partial A_Y}{\partial Y} + \frac{\partial A_Z}{\partial Z} \right) + A_X \frac{\partial \omega_0}{\partial X} + A_Y \frac{\partial \omega_0}{\partial Y} + A_Z \frac{\partial \omega_0}{\partial Z} = -\rho / \epsilon_0 \quad -(93)$$

Eqs. (82) to (84), (88) to (90), and (93) are seven equations in seven unknowns:

$$A_X, A_Y, A_Z, \omega_0, \omega_X, \omega_Y, \omega_Z \quad -(94)$$

given that ρ / ϵ_0 is known experimentally.

So the static electric flux density can be found in general by solving these equations.

An example solution can be found by assuming the Coulomb field, which is one of the most accurately tested laws in physics:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{e_2}{4\pi\epsilon_0 r^3}\underline{r} \,. \quad -(95)$$

This has the solution

$$\phi = -\frac{e_2}{8\pi\epsilon_0 r}, \quad \underline{\omega} = \frac{\underline{r}}{r^2}, \quad -(96)$$

$$\underline{\nabla}\phi = \frac{e_2}{8\pi\epsilon_0 r^3}\underline{r}, \quad \underline{\omega}\phi = -\frac{e_2}{8\pi\epsilon_0 r^3}\underline{r}, \quad -(97)$$

so the electric field strength is proportional to $\underline{A}$:

$$\underline{E} = -\omega_0 \underline{A} = -\frac{e_2}{4\pi\epsilon_0 r^3}\underline{r} \,. \quad -(98)$$

From Eq. (80):

$$\underline{A}\frac{\partial \omega_0}{\partial t} = \underline{0} \quad -(99)$$

so the scalar spin connection is time independent:

$$\frac{\partial \omega_0}{\partial t} = 0 \,. \quad -(100)$$

From Eq. (78):

$$\underline{\nabla} \times (\omega_0 \underline{A}) = \omega_0 \underline{\nabla} \times \underline{A} + \underline{A} \times \underline{\nabla}\omega_0 = \underline{0} \,. \quad -(101)$$

Using:

$$\underline{A} = \frac{e_2}{4\pi\epsilon_0 \omega_0}\frac{\underline{r}}{r^3} \quad -(102)$$

it follows that:

$$\omega_0 \underline{\nabla} \times \underline{A} = \underline{0} \quad -(103)$$

so

$$\underline{A} \times \underline{\nabla} \omega_0 = \underline{0}. \quad -(104)$$

A possible solution is:

$$\underline{A} = \frac{e_2}{4\pi\epsilon_0\omega_0 c} \underline{\nabla} \omega_0 \quad -(105)$$

so

$$\underline{\nabla} \omega_0 = c \frac{\underline{r}}{r^3} \quad -(106)$$

and

$$\omega_0 = -\frac{c}{r}. \quad -(107)$$

The complete solution for the static electric field strength is

$$\underline{E} = -\omega_0 \underline{A} = -\frac{e_2}{4\pi\epsilon_0 r^3} \underline{r}, \quad -(108)$$

$$\omega_0 = -c/r, \quad \underline{\omega} = \underline{r}/r^3, \quad -(109)$$

$$\phi = -\frac{e_2}{8\pi\epsilon_0 r}, \quad \underline{A} = -\frac{e_2}{4\pi\epsilon_0 r^2 c} \underline{r}, \quad -(110)$$

$$\partial \underline{A} / \partial t = \underline{0}. \quad -(111)$$

As shown in detail in Note 381(5) a solution of identical structure exists for the gravitostatic field equations of ECE2:

$$\underline{g} = -\underline{\nabla}\underline{\Phi} + \underline{\omega}\,\underline{\Phi} = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q}, \quad Q^\mu = \left(\frac{\Phi}{c}, \underline{Q}\right) \quad -(112)$$

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(113)$$

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(114)$$

$$\partial \underline{g} / \partial t = \underline{0} \quad -(115)$$

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} = \underline{0} \quad -(116)$$

where Φ is the gravitational scalar potential, $\underline{Q}$ is the gravitational vector potential, Q^μ is the gravitational four vector:

$$Q^\mu = \left(\frac{\Phi}{c}, \underline{Q}\right), \quad -(117)$$

$\underline{g}$ is the gravitostatic field, $\underline{\Omega}$ is the gravitomagnetic field, G is Newton's constant and ρ_m is the source mass density. The relevant S. I. Units are as follows:

$$\begin{aligned} g &= m s^{-2} \\ \Phi &= m^2 s^{-2}, \quad \underline{Q} = m s^{-1}, \\ \omega_0 &= s^{-1} \end{aligned} \quad -(118)$$

The complete solution for ECE2 gravitostatics is:

$$\underline{g} = -\omega_0 \underline{Q} = -\frac{mG}{r^3}\underline{r} \quad -(119)$$

$$\omega_0 = -c/r, \quad \underline{\omega} = \underline{r}/r^3 \quad -(120)$$

$$\Phi = -\frac{mG}{2r}, \quad \underline{Q} = -\frac{mG}{cr^2}\underline{r} \quad -(121)$$

$$\partial \underline{Q} / \partial t = \underline{0} \quad -(122).$$

Finally, Notes 381(6) and 381(7) check that the antisymmetry laws are obeyed.

In ECE2 electrostatics for example:

$$\underline{\omega} = \underline{r}/r^2, \quad \underline{A} = -\frac{e_2}{4\pi\epsilon_0 c}\frac{\underline{r}}{r^2} \quad -(123)$$

so:

$$\underline{\omega} \times \underline{A} = \underline{0}. \quad -(124)$$

In component format:

$$\omega_Y A_Z - \omega_Z A_Y = 0 \quad -(125)$$

$$\omega_Z A_X - \omega_X A_Z = 0 \quad -(126)$$

$$\omega_X A_Y - \omega_Y A_X = 0. \quad -(127)$$

From Eq. (123):

$$A_X = -\frac{e_2}{4\pi\epsilon_0 c} \frac{X}{(X^2+Y^2+Z^2)} \quad -(128)$$

$$A_Y = -\frac{e_2}{4\pi\epsilon_0 c} \frac{Y}{(X^2+Y^2+Z^2)} \quad -(129)$$

so:

$$\partial A_X / \partial Y = \frac{e_2}{2\pi\epsilon_0 c} \frac{XY}{(X^2+Y^2+Z^2)^2} \quad -(130)$$

$$\partial A_Y / \partial X = \frac{e_2}{2\pi\epsilon_0 c} \frac{YX}{(X^2+Y^2+Z^2)^2} \quad -(131)$$

and:

$$\frac{\partial A_X}{\partial Y} = \frac{\partial A_Y}{\partial X} \quad -(132)$$

et cyclicum. Therefore the antisymmetry laws for A are obeyed:

$$\frac{\partial A_X}{\partial Y} - \frac{\partial A_Y}{\partial X} = \omega_X A_Y - \omega_Y A_X = 0 \quad -(133)$$

et cyclicum.

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