GENERAL RELATIVITY AND COSMOLOGY WITHOUT THE METRIC

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ABSTRACT

It is shown that general relativity and cosmology can be developed without the use of the metric by expressing the Riemann tensor as the covariant derivative of a rank three tensor denoted $S^{\mu}_{\rho\sigma}$. In the weak field limit, Newtonian dynamics are recovered and the acceleration due to gravity, $\underline{g}$, is shown to be $c^2 \underline{S}_{00}$. The method is shown to be a limiting case of the Einstein Cartan Evans (ECE) field theory, which is generally covariant in all its sectors.

Keywords: ECE field theory, general relativity without the metric, Newtonian limit.

91st paper of ECE Theory

1. INTRODUCTION

It is well known that the Newtonian limit of gravitational general relativity is obtained from the Einstein Hilbert field equation by considering the weak field limit. This is the limit of a quasi-static field and slow moving particles. In so doing, the Christoffel symbol is expressed in terms of a symmetric metric using the metric compatibility condition {1}. The development of general relativity the proceeds in terms of models for the metric. However, Crothers {2} has shown clearly that this development is flawed geometrically at a fundamental level, so that concepts such as Big Bang, black holes and dark matter do not exist in nature. The Einstein Cartan Evans (ECE) field theory {3-10} has explained dark matter and the dynamics of spiral galaxies, for example, in terms of the Cartan torsion. The ECE method does repeat the elementary errors pointed out by Crothers {2} in standard cosmology. These are basic geometrical errors which have been uncritically repeated, especially in regard to singularities. The latter were not introduced by Einstein himself and can all be traced to the use of the metric.

In Section 2 the Riemann tensor is expressed as the covariant derivative of a rank three tensor denoted $S^{\mu}{}_{\rho\sigma}$:

$$R^{\mu}{}_{\sigma\nu\rho} := D_{\nu} S^{\mu}{}_{\rho\sigma}. \qquad -(1)$$

Dynamical solutions which result directly from this definition are used to find the Newtonian limit without the use of the metric. It is shown that the Newtonian acceleration due to gravity is:

$$\underline{g} = c^2 \, \underline{S}_{00} \qquad -(2)$$

where c is the vacuum speed of light. For comparison, the method used by Einstein is given, a method which expresses the Christoffel in terms of the metric. This is a more complicated method, so by Ockham's Razor, the method based on Eq. (1) is preferred. The latter method by passes the errors pointed out by Crothers {2} and also gives novel dynamical information based on constraints on the gamma connection. Finally it is shown that the method of Eq. (1) is a special case of ECE theory.

2. NEWTONIAN LIMIT

Consider the Riemann tensor expressed as:

$$R^{\mu}{}_{\sigma\nu\rho} := D_{\nu} S^{\mu}{}_{\rho\sigma} . \qquad -(3)$$

The rule {1} for the covariant derivative of a tensor of any rank is:

$$\begin{aligned} D_{\sigma} V^{\mu_1 \mu_2 \cdots \mu_k}{}_{\nu_1 \nu_2 \cdots \nu_l} &= \partial_{\sigma} V^{\mu_1 \mu_2 \cdots \mu_k}{}_{\nu_1 \nu_2 \cdots \nu_l} \\ &+ \Gamma^{\mu_1}{}_{\sigma\lambda} V^{\lambda \mu_2 \cdots \mu_k}{}_{\nu_1 \nu_2 \cdots \nu_l} + \Gamma^{\mu_2}{}_{\sigma\lambda} V^{\mu_1 \lambda \cdots \mu_k}{}_{\nu_1 \nu_2 \cdots \nu_l} + \cdots \\ &- \Gamma^{\lambda}{}_{\sigma\nu_1} V^{\mu_1 \mu_2 \cdots \mu_k}{}_{\lambda \nu_2 \cdots \nu_l} - \Gamma^{\lambda}{}_{\sigma\nu_2} V^{\mu_1 \mu_2 \cdots \mu_k}{}_{\nu_1 \lambda \cdots \nu_l} - \cdots \end{aligned} \qquad -(4)$$

For example:

$$D_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}{}_{\lambda\mu} V^{\lambda} , \qquad -(5)$$

$$D_{\mu} V_{\nu} = \partial_{\mu} V_{\nu} - \Gamma^{\lambda}{}_{\nu\mu} V_{\lambda} , \qquad -(6)$$

$$D_{\mu} V^{\nu\rho} = \partial_{\mu} V^{\nu\rho} + \Gamma^{\nu}{}_{\mu\lambda} V^{\lambda\rho} + \Gamma^{\rho}{}_{\mu\lambda} V^{\nu\lambda} . \qquad -(7)$$

So:

$$D_{\nu} S^{\mu}{}_{\rho\sigma} = \partial_{\nu} S^{\mu}{}_{\rho\sigma} + \Gamma^{\mu}{}_{\nu\lambda} S^{\lambda}{}_{\rho\sigma} - \Gamma^{\lambda}{}_{\nu\rho} S^{\mu}{}_{\lambda\sigma} - \Gamma^{\lambda}{}_{\nu\sigma} S^{\mu}{}_{\rho\lambda} . \qquad -(8)$$

Compare Eq. (8) with the definition of the Riemann tensor:

$$R^{\mu}{}_{\sigma\nu\rho} = \partial_\nu \Gamma^{\mu}_{\rho\sigma} - \partial_\rho \Gamma^{\mu}_{\nu\sigma} + \Gamma^{\mu}_{\nu\lambda}\Gamma^{\lambda}_{\rho\sigma} - \Gamma^{\mu}_{\rho\lambda}\Gamma^{\lambda}_{\nu\sigma}. \quad -(9)$$

Comparing Eqs. (8) and (9) it is seen that they are the same if:

$$\Gamma^{\mu}_{\rho\lambda}\Gamma^{\lambda}_{\nu\sigma} = \Gamma^{\lambda}_{\nu\sigma} S^{\mu}_{\rho\lambda}, \quad -(10)$$

$$\Gamma^{\mu}_{\nu\lambda}\Gamma^{\lambda}_{\rho\sigma} = \Gamma^{\mu}_{\nu\lambda} S^{\lambda}_{\rho\sigma}, \quad -(11)$$

$$\partial_\nu \Gamma^{\mu}_{\rho\sigma} = \partial_\nu S^{\mu}_{\rho\sigma}, \quad -(12)$$

$$\partial_\rho \Gamma^{\mu}_{\nu\sigma} = \Gamma^{\lambda}_{\nu\rho} S^{\mu}_{\lambda\sigma}. \quad -(13)$$

Therefore it is found that:

$$S^{\lambda}_{\rho\sigma} = \Gamma^{\lambda}_{\rho\sigma} \quad -(14)$$

if

$$\partial_\rho \Gamma^{\mu}_{\nu\sigma} = \Gamma^{\lambda}_{\nu\rho}\Gamma^{\mu}_{\lambda\sigma}. \quad -(15)$$

Eq. (14) identifies $S^{\lambda}_{\rho\sigma}$ with the gamma connection provided the latter is constrained by Eq. (15).

In the Einstein Hilbert (EH) limit:

$$\Gamma^{\lambda}_{\rho\sigma} = \Gamma^{\lambda}_{\sigma\rho} \quad -(16)$$

which is the Christoffel symbol. In this limit there is no Cartan torsion {3-8}. The EH limit reduces to Newtonian dynamics {1} by reducing the EH field equation to the Poisson equation:

$$\nabla^2 \Phi = 4\pi G \rho \quad -(17)$$

where G is Newton's constant, Φ is the gravitational potential and ρ is the mass density. Firstly in this section, the method used by Einstein is reviewed, and secondly it is shown that it is simpler and clearer to obtain the Newtonian limit from Eq. (1) without using the metric. So no models for the metric are needed in cosmology. It is these models that are flawed fundamentally as shown by Crothers {2}.

Einstein's method {1} of obtaining the Newtonian limit starts from his field equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad -(18)$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the scalar curvature, $g_{\mu\nu}$ the symmetric metric, k is Einstein's constant:

$$k = \frac{8\pi G}{c^2} \quad -(19)$$

and $T_{\mu\nu}$ is the symmetric canonical energy momentum tensor of Noether. The equation (18) can be contracted to:

$$R = -kT \quad -(20)$$

which gives:

$$R_{\mu\nu} = k\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right). \quad -(21)$$

The Newtonian limit is the weak field, quasi-static, slow moving particle limit. In this limit $T_{\mu\nu}$ is dominated by the rest energy, so:

$$T_{00} = \rho = \frac{m}{V} \quad -(22)$$

where m is the particle mass and V a volume. In this limit the metric is a perturbation of the flat space Minkowski metric:

$$g_{00} = -1 + h_{00} \quad -(23)$$

$$g^{00} = -1 - h^{00} \quad -(24)$$

so

$$T = g^{00} T_{00} = -T_{00}. \quad -(25)$$

Therefore from Eq. (21):

$$R_{00} = R^{\lambda}{}_{0\lambda 0} = \frac{1}{2} k T_{00} \quad -(26)$$

because:

$$R^{0}{}_{000} = 0. \quad -(27)$$

Einstein therefore considered:

$$R^{i}{}_{0j0} = \partial_j \Gamma^{i}_{00} - \partial_0 \Gamma^{i}_{j0} + \Gamma^{i}_{j\lambda} \Gamma^{\lambda}_{00} - \Gamma^{i}_{0\lambda} \Gamma^{\lambda}_{j0} \quad -(28)$$

and approximated this by:

$$R^{i}{}_{0j0} = \partial_j \Gamma^{i}_{00} \quad -(29)$$

by neglecting terms second order in the connection and assuming a quasi static field, so:

$$\partial_0 \Gamma^{i}_{j0} = 0. \quad -(30)$$

The Ricci tensor element of relevance is then:

$$R_{00} = R^{i}{}_{0i0} = \partial_i \left(\frac{1}{2} g^{i\lambda} \left(\partial_0 g_{\lambda 0} + \partial_0 g_{0\lambda} - \partial_\lambda g_{00} \right) \right) \quad -(31)$$

where the Christoffel symbol has been expressed in terms of metric components using the metric compatibility condition {1}. In Eq. (31), time derivatives are again neglected, so in this approximation:

$$R_{00} = -\frac{1}{2}\partial_i g^{i\lambda}\partial_\lambda g_{00} \qquad -(32)$$

$$= -\frac{1}{2}\nabla^2 h_{00} = \frac{1}{2}kT_{00} = \frac{1}{2}k\rho .$$

Thus

$$\nabla^2 h_{00} = -kT_{00} \qquad -(33)$$

which relates a component of the Ricci tensor to the rest energy or mass density. Therefore:

$$\nabla^2 h_{00} = -k\rho \qquad -(34)$$

and this is the Poisson equation (17) if:

$$h_{00} = -\frac{2}{c^2}\Phi \qquad -(35)$$

Q.E.D.

Using this well known derivation by Einstein as a guide line, it is now shown that the derivation of the Newtonian limit from Eq. (1) is simpler and does not use the metric at all. So problems {2} generated by modeling the metric can be eliminated from general relativity and cosmology. In particular, unphysical singularities are eliminated.

In the weak field limit, terms to second order in the connection in Eq. (1) are neglected, so:

$$D_\sigma S^\mu{}_{\nu\rho} \sim \partial_\sigma S^\mu{}_{\nu\rho} = R^\mu{}_{\sigma\nu\rho} . \qquad -(36)$$

Thus:

$$\partial_j S^i_{\ 00} = R^i_{\ 0j0} \qquad -(37)$$

and

$$\partial_i S^i_{\ 00} = R^i_{\ 0i0} = R_{00} = \frac{1}{2} k T_{00} = \frac{1}{2} k \rho . \qquad -(38)$$

Now compare:

$$\partial_i S^i_{\ 00} = \frac{1}{2} k \rho \qquad -(39)$$

with the Poisson equation (17), using:

$$\nabla^2 = -\partial_i \partial^i . \qquad -(40)$$

Eq. (17) is:

$$-\partial_i \partial^i \Phi = 4\pi G \rho \qquad -(41)$$

so:

$$S^i_{\ 00} = -\frac{1}{c^2} \partial^i \Phi . \qquad -(42)$$

In vector notation:

$$\underline{S}_{00} = -\frac{1}{c^2} \underline{\nabla} \Phi . \qquad -(43)$$

However we know that:

$$\underline{g} = -\underline{\nabla} \Phi \qquad -(44)$$

where $\underline{g}$ is the acceleration due to gravity, so:

$$\underline{g} = c^2 \underline{S}_{00}. \quad -(45)$$

So we have obtained the Poisson equation and Newton inverse square law directly from the definition (1) without using the metric, Q.E.D. In so doing the constraint (15) has been inferred, and this contains new dynamical information.

The EH field equation can be expressed as:

$$D_\kappa R^\mu{}_{\sigma\nu\rho} + D_\kappa R^\mu{}_{\rho\sigma\nu} + D_\kappa R^\mu{}_{\nu\rho\sigma} = k\left(D_\kappa T^\mu{}_{\sigma\nu\rho} + D_\kappa T^\mu{}_{\rho\sigma\nu} + D_\kappa T^\mu{}_{\nu\rho\sigma}\right). \quad -(46)$$

This is equivalent to Eq. (18) as can be shown {1} by index contraction. In the EH limit:

$$R^\mu{}_{\sigma\nu\rho} + R^\mu{}_{\rho\sigma\nu} + R^\mu{}_{\nu\rho\sigma} = D_\nu S^\mu{}_{\rho\sigma} + D_\sigma S^\mu{}_{\nu\rho} + D_\rho S^\mu{}_{\sigma\nu} = 0. \quad -(47)$$

In differential form notation, Eq. (47) is:

$$R^a{}_b \wedge q^b = (DS)^a{}_b \wedge q^b = 0 \quad -(48)$$

where in this notation:

$$(DS)^a{}_b := D_\nu S^a_\rho{}_b. \quad -(49)$$

More generally {1, 3-8} the Cartan torsion must be considered, so

$$(DS)^a{}_b \wedge q^b := D \wedge T^a. \quad -(50)$$

This is the Bianchi identity of differential geometry, there being only one fundamental Bianchi identity which can therefore be expressed in terms of S and T.

In conclusion therefore this method eliminates the metric from general relativity, and eliminates the fatal flaws in standard cosmology pointed out by Crothers. Future work will develop general relativity and cosmology in terms of S, without use of the metric, the first step being to re-work Einstein's well known calculation of light deflection due to gravity.

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