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On the Violation of the Bianchi Identity by the Einstein Field Equation and Big Bang Cosmologies

by

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Abstract

It is shown that the Bianchi identity and its Hodge dual identity are cyclic sums of curvature tensors identically equal to the cyclic sums of the definitions of the same curvature tensors. In consequence, the torsion tensor cannot be neglected or assumed to be zero. This is a key result that shows the fundamental incorrectness of the Einstein field equation and all big bang cosmologies. The correct field equations of cosmology must be based directly on the Bianchi identity with finite torsion and curvature.

Keywords: Violation of the Bianchi identity by the Einstein field equation, incorrectness of big bang cosmologies, ECE equations of dynamics.

19.1 Introduction

The Bianchi identity of Cartan geometry [1–12] is:

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{19.1}$$

in the notation of differential geometry [1]. Here $D\wedge$ is the covariant exterior derivative, T^a is the torsion form, $R^a{}_b$ is the curvature form, $\wedge$ denoted wedge product, and q^b is the tetrad form. This identity is the homogeneous field equation of dynamics in ECE theory. Eq. (19.1) implies that there exists the identity:

$$D \wedge \widetilde{T}^a := \widetilde{R}^a{}_b \wedge q^b \tag{19.2}$$

where the tilde denotes Hodge duality in four dimensions. Eq. (19.2) is the inhomogeneous field equation of dynamics in ECE theory. It is seen that the equations of dynamics in ECE field theory are duality invariant, a fundamental symmetry property. In Section 2 it is proven that Eqs. (19.1) and (19.2) in tensor notation are both cyclic sums of curvature tensors identically equal to the same cyclic sum of definitions of the same curvature tensors. In order to arrive at this result the tetrad postulate is used:

$$D_\mu q^a_\nu = 0 \tag{19.3}$$

and the torsion tensor must be identically non-zero. The proof of Eq. (19.2) from Eq. (19.1) is therefore straightforward, because the Hodge dual of the curvature tensor in four dimensions is another curvature tensor. The cyclic sum of the latter is identically equal again to the sum of definitions, QED. Various other simple proofs of Eq. (19.2) are given in this section. All solutions of the Einstein field equation in the presence of finite canonical energy momentum density give the result:

$$D \wedge \widetilde{T}^a = 0 \tag{19.4}$$

and the result:

$$\widetilde{R}^a{}_b \wedge q^b \neq 0 \tag{19.5}$$

and so the Einstein field equation violates the Bianchi identity in its form (19.2). No physical inference of any kind can be drawn from the Einstein field equation because of its neglect of torsion. The only exception is the class of Ricci flat solutions of the Einstein field equation. Computer algebra showed that simple members of this class fortuitously obey both equations (19.1) and (19.2), but as shown by Crothers and independently elsewhere [1–12], vacuum solutions have no physical meaning. They are just equations of geometry that assume a priori that the Ricci tensor vanishes. The era of the great Einstein field equation has therefore drawn to a close. The correct field equations are derived directly from Eqs. (19.1) and (19.2) and are respectively the homogeneous and inhomogeneous dynamical equations of Einstein Cartan Evans (ECE) field theory [1–12]. The electro-dynamical field equations of ECE theory have the same structure as the dynamical field equations.

19.2 The Tensorial Formats of the Bianchi Identity

Expanding Eq. (19.1) using the well known [1–12] definitions of $D\wedge$ and of the wedge product, the following equation is obtained:

$$D_\mu T^a_{\nu\rho} + D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} := R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} \tag{19.6}$$

in which appears the curvature and torsion tensors, and the spin connection. By considering the well known [1–12] action of the commutator of covariant derivatives on a four-vector the curvature and torsion tensors are obtained as follows:

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \tag{19.7}$$

irrespective of any metric and of any connection. There is no reason why the torsion tensor should be zero a priori, as incorrectly assumed in the standard model and big bang cosmologies. The two tensors are defined by the commutator, [1–12], and are:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\mu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \tag{19.8}$$

and

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{19.9}$$

where $T^\lambda_{\mu\nu}$ is the connection defined by the covariant derivative:

$$D_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda} V^\lambda. \tag{19.10}$$

The standard model of physics, and big bang cosmologies, incorrectly assume that the connection is symmetric:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} \tag{19.11}$$

and that in consequence the torsion vanishes. As argued already, this assumption leads to a violation of the Bianchi identity, i.e. to an incorrect geometry. This basic flaw in the Einstein equation was discovered by computer algebra in papers 93 ff of www.aias.us. It was obviously not known to Einstein in 1915 and for ninety two years thereafter.

The connection defined in Eq. (19.10) is related to the spin connection by the tetrad postulate [1–12]:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu - q^a_\lambda \Gamma^\lambda_{\mu\nu} + q^b_\nu \omega^a_{\mu b} = 0. \tag{19.12}$$

There is no situation in physics in which the tetrad postulate is not true, because the postulate is the fact that a complete vector field is independent of the coordinate system used to define the vector field. The curvature and torsion forms and their respective tensors are related by the tetrad [1–12] as follows:

$$T^{\lambda}_{\mu\nu} = q^{\lambda}_{a} T^{a}_{\mu\nu}, \tag{19.13}$$

$$R^{\rho}{}_{\sigma\mu\nu} = q^{\rho}_{a} q^{b}_{\sigma} R^{a}{}_{b\mu\nu}. \tag{19.14}$$

Using Eqs. (19.8) to (19.14) it follows [1–12] that the Bianchi identity (19.1) is:

$$\begin{aligned} &R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\nu\rho\mu} \\ &:= \partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho} \\ &\quad + \partial_\nu \Gamma^{\lambda}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu} \\ &\quad + \partial_\rho \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} \neq 0 \end{aligned} \tag{19.15}$$

where:

$$\begin{aligned} R^{\lambda}{}_{\rho\mu\nu} &= \partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho}, \\ R^{\lambda}{}_{\mu\nu\rho} &= \partial_\nu \Gamma^{\lambda}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu}, \\ R^{\lambda}{}_{\nu\rho\mu} &= \partial_\rho \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu}. \end{aligned} \tag{19.16}$$

Therefore the Bianchi identity is a true identity, one side is identically the same as the other by definition. This result of Cartan geometry is true if and only if the torsion tensor is identically non-zero. Unfortunately, the standard model literature uses the equation:

$$R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu} = 0 \tag{19.17}$$

and names it the "first Bianchi identity". From the foregoing it is clear that Eq. (19.17) is true only if the connection is a priori assumed to be symmetric as in Eq. (19.11). There is no basis of this assumption. This error of the standard model is compounded by its use of the equation:

$$D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu} = 0 \tag{19.18}$$

which is referred to as "the second Bianchi identity". Unfortunately Eq. (19.18) again neglects torsion and again leads [1–12] to an incorrect geometry. There is therefore a catastrophic failure of the standard model because

Einstein based his field equation directly on Eq. (19.18) as is well known. The true Bianchi identity is Eq. (19.1), which in tensorial format is Eq. (19.15).

The easiest way to prove Eq. (19.2) from Eq. (19.1) is to consider the Hodge dual of the curvature form in four dimensions. This Hodge dual is well known [1–2] to be defined as:

$$\widetilde{R}^{\rho}{}_{\sigma}{}^{\alpha\beta} = \frac{1}{2} \parallel g \parallel^{\frac{1}{2}} \epsilon^{\mu\nu\alpha\beta} R^{\rho}{}_{\sigma\mu\nu} \tag{19.19}$$

where $\parallel g \parallel^{\frac{1}{2}}$ is the square root of the determinant of the metric and where $\epsilon^{\mu\nu\alpha\beta}$ is the totally anti-symmetric unit tensor. The latter is defined to be the Minkowski space-time tensor [1–12]. The Hodge dual of the curvature form is therefore another curvature form, the cyclic sum of which obeys the equation (19.15) again, Q.E.D. The catastrophic failure of the standard model of cosmology emerged in paper 93 (www.aias.us) when considering the Hodge dual Bianchi identity (19.2) in the tensor format:

$$D_\mu \widetilde{T}^{\kappa}_{\nu\rho} + D_\rho \widetilde{T}^{\kappa}_{\mu\nu} + D_\nu \widetilde{T}^{\kappa}_{\rho\mu} := \widetilde{R}^{\kappa}{}_{\mu\nu\rho} + \widetilde{R}^{\kappa}{}_{\rho\mu\nu} + \widetilde{R}^{\kappa}{}_{\nu\rho\mu}. \tag{19.20}$$

This equation is the same as:

$$D_\mu T^{\kappa\mu\nu} := R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{19.21}$$

It was found by computer algebra in paper 93 ff. of www.aias.us that the Einstein field equation violates Eq. (19.21) because the Einstein field equation gives the result:

$$T^{\kappa\mu\nu} = 0, R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0 \tag{19.22}$$

in general. For example the result (19.22) was obtained for the Robertson Walker metric which is the basis for big bang. No big bang metric survived the test of Eq. (19.21), the inhomogeneous field equation of ECE cosmology.

This disaster for the standard model remained hidden for so many years because the Einstein field equation fortuitously obeys the homogeneous ECE field equation, whose tensorial format is:

$$D_\mu T^{\kappa}_{\nu\rho} + D_\rho T^{\kappa}_{\mu\nu} + D_\nu T^{\kappa}_{\rho\mu} := R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\nu\rho\mu} \tag{19.23}$$

which is the same as:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} := \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{19.24}$$

The standard model has assumed for over ninety years that:

$$\widetilde{T}^{\kappa\mu\nu} = 0, \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} = 0. \tag{19.25}$$

In the rest of this section we give simple proofs of Eq. (19.2) from Eq. (19.1). Given the tetrad postulate, the Bianchi identity as we have argued can be constructed from commutator relations such as:

$$\begin{aligned}[D_0, D_1]V^\rho &= D_0(D_1 V^\rho) - D_1(D_0 V^\rho) \\ &= R^\rho{}_{\sigma 01} V^\sigma - T^\lambda_{01} D_\lambda V^\rho.\end{aligned} \tag{19.26}$$

The Hodge dual of Eq. (19.26) with lowered indices is:

$$[D_2, D_3]V^\rho = D_2(D_3 V^\rho) - D_3(D_2 V^\rho) = R^\rho{}_{\sigma 23} V^\sigma - T^\lambda_{23} D_\lambda V^\rho. \tag{19.27}$$

Equations (19.26) and (19.27) are both examples of:

$$[D_\mu, D_\nu]V^\rho = D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho). \tag{19.28}$$

So it follows that:

$$D \wedge \widetilde{T}^a := \widetilde{R}^a{}_b \wedge q^b \tag{19.29}$$

is an example of

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{19.30}$$

Q.E.D.

This is a simple proof given in outline. Some more details can be given as follows. Carrying out Hodge duals on both sides of Eq. (19.26):

$$\epsilon^{0123}[D_0, D_1]V^\rho = \epsilon^{0123} R^\rho{}_{\sigma 01} V^\sigma - \epsilon^{0123}\ T^\lambda_{01} D_\lambda V^\rho \tag{19.31}$$

i.e.:

$$[D^2, D^3]V^\rho = R^\rho{}_\sigma{}^{23} V^\sigma - T^{\lambda 23} D_\lambda V^\rho. \tag{19.32}$$

Indices are lowered as follows:

$$[D_2, D_3] = g_{22}\ g_{33}[D^2, D^3] \tag{19.33}$$

on both sides, giving Eq. (19.27). Therefore if

$$\mu = 0, \nu = 1 \tag{19.34}$$

in the original identity, then:

$$\mu = 2, \nu = 3 \tag{19.35}$$

in the Hodge dual identity. Both the original and Hodge dual identities are cyclic sums of curvature tensors as discussed already.

It is helpful to give examples of the results of paper 93. It is clear that these are technically correct because they were generated by computer. It was found that for exact solutions of the Einstein field equation with finite canonical energy-momentum density:

$$T_{\mu\nu} \neq 0 \tag{19.36}$$

the key curvature tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ was non-zero. For example:

$$R^{0}{}_{1}{}^{01} + R^{0}{}_{2}{}^{02} + R^{0}{}_{3}{}^{03} \neq 0. \tag{19.37}$$

These exact solutions use the Christoffel connection, so by definition the solutions assume that the torsion tensor vanishes. Thus for example:

$$T^{001} + T^{002} + T^{003} = 0. \tag{19.38}$$

Therefore:

$$D_1 T^{001} + D_2 T^{002} + D_3 T^{003} = 0. \tag{19.39}$$

Therefore the Hodge dual of the Bianchi identity is not obeyed:

$$D_1 T^{001} + D_2 T^{002} + D_3 T^{003} \neq R^{0}{}_{1}{}^{01} + R^{0}{}_{2}{}^{02} + R^{0}{}_{3}{}^{03} \tag{19.40}$$

a fundamental error of the Einstein field equation which is the basis of all current cosmologies.

Using the Hodge dual relations:

$$\begin{aligned} \widetilde{R}^{0}{}_{123} &= \epsilon_{0123} R^{0}{}_{1}{}^{01}, \\ \widetilde{R}^{0}{}_{312} &= \epsilon_{0312} R^{0}{}_{3}{}^{03}, \\ \widetilde{R}^{0}{}_{231} &= \epsilon_{0231} R^{0}{}_{2}{}^{02}, \end{aligned} \tag{19.41}$$

it is found that:

$$\widetilde{R}^0{}_{123} + \widetilde{R}^0{}_{312} + \widetilde{R}^0{}_{231} \neq 0 \tag{19.42}$$

for all solutions of the Einstein field equation with finite $T_{\mu\nu}$. These are the type of solutions used in big bang cosmology. This result is technically irrefutable because it was generated by computer algebra. The latter also showed in paper 93 of www.aias.us that:

$$R^0{}_{123} + R^0{}_{312} + R^0{}_{231} = 0 \tag{19.43}$$

for all correct solutions of the Einstein field equation. Eq. (19.43) is an example of what is incorrectly referred to in the standard model as "the first Bianchi identity". The Hodge dual of Eq. (19.43) is:

$$\widetilde{R}^0{}_1{}^{01} + \widetilde{R}^0{}_2{}^{02} + \widetilde{R}^0{}_3{}^{03} = 0. \tag{19.44}$$

So the Bianchi identity:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} := \widetilde{R}^\kappa{}_\mu{}^{\mu\nu} \tag{19.45}$$

is obeyed fortuitously, but the Hodge dual:

$$D_\mu T^{\kappa\mu\nu} := R^\kappa{}_\mu{}^{\mu\nu} \tag{19.46}$$

is not obeyed. This result was first shown in paper 93 of www.aias.us. It is difficult to find because the calculations needed are far to complicated to be carried out by hand. They need the development of powerful Maxima based code.

These are examples of the general theorem:

$$\begin{aligned} [D_\mu, D_\nu]\, V^\rho &= D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho) \\ &= R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \end{aligned} \tag{19.47}$$

which is the result of the anti-symmetry of the commutator:

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] \tag{19.48}$$

of covariant derivatives. This anti-symmetry defines the anti-symmetry in the last two indices of the curvature tensor:

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu} \tag{19.49}$$

and the torsion tensor:

$$T^{\lambda}_{\mu\nu} = -T^{\lambda}_{\nu\mu}. \tag{19.50}$$

The catastrophic situation has arisen that the incorrect assumption:

$$T^{\lambda}_{\mu\nu} = 0 \tag{19.51}$$

of Einstein has worked its way uncritically into the subject of cosmology for ninety years. The assumption of vanishing torsion was shown in paper 93 of www.aias.us to violate the complete Bianchi identity. In order to show this, the Hodge dual of the identity has to be taken into account. The resulting calculations are so complicated that computer algebra must be used throughout. Correct cosmology must be based on a computer solution of the Bianchi identity simultaneously with its Hodge dual. In ECE theory these are the equations of relativistic dynamics and cosmology. In so doing the Christoffel connection cannot be used, the connection is in general not symmetric in its lower two indices. So the well known equation that links the Christoffel connection to the symmetric metric:

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \tag{19.52}$$

must be discarded, because this leads to a violation of the Bianchi identity. The latter must always be written with non-zero torsion. As a result of this major change in cosmology it has been shown in ECE papers on www.aias.us that there exist relations such as:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 T \tag{19.53}$$

where $\boldsymbol{g}$ is the acceleration due to gravity and where T is a well defined scalar torsion. It is found that the acceleration due to gravity $\boldsymbol{g}$ and the electric field strength $\boldsymbol{E}$ are both due to space-time torsion. The basic structure of spiral galaxies is also due to space-time torsion, so maps of the so called "dark matter" are maps of space-time torsion

The Hodge dual of the commutator operator is another commutator operator:

$$[D^\alpha, D^\beta]_{HD} = \frac{1}{2} \parallel g \parallel^{\frac{1}{2}} \epsilon^{\mu\nu\alpha\beta}[D_\mu, D_\nu] \tag{19.54}$$

and it follows that:

$$[D^\alpha, D^\beta]_{HD} V^\sigma = \widetilde{R}^{\rho}{}_{\sigma}{}^{\alpha\beta} V^\sigma - \widetilde{T}^{\lambda\alpha\beta} D_\lambda V^\rho \tag{19.55}$$

where:

$$\widetilde{R}^{\rho}{}_{\sigma}{}^{\alpha\beta} = \frac{1}{2} \parallel g \parallel^{\frac{1}{2}} \epsilon^{\mu\nu\alpha\beta} R^{\rho}_{\sigma\mu\nu}. \tag{19.56}$$

and

$$\widetilde{T}^{\lambda\alpha\beta} = \frac{1}{2} \parallel g \parallel^{\frac{1}{2}} \epsilon^{\mu\nu\alpha\beta} T^{\lambda}_{\mu\nu}. \tag{19.57}$$

In general, indices are lowered by:

$$\begin{aligned} [D_\alpha, D_\beta]_{HD} &= g_{\alpha\mu} g_{\beta\nu} [D^\mu, D^\nu]_{HD} \\ \widetilde{R}^{\rho}{}_{\sigma\alpha\beta} &= g_{\alpha\mu}\ g_{\beta\nu} \widetilde{R}^{\rho}{}_{\sigma}{}^{\mu\nu} \\ \widetilde{T}^{\lambda}_{\alpha\beta} &= g_{\alpha\mu}\ g_{\beta\nu} \widetilde{T}^{\lambda\alpha\beta} \end{aligned} \tag{19.58}$$

so it is found that:

$$[D_\alpha, D_\beta]_{HD} V^\sigma = \widetilde{R}^{\rho}{}_{\alpha\beta} V^\sigma - \widetilde{T}^{\lambda}_{\alpha\beta} D_\lambda V^\rho. \tag{19.59}$$

This is the Hodge dual with lowered indices of Eq. (19.47). The Hodge duals of the curvature and torsion tensors are curvature and torsion tensors with different indices. So it follows that they obey the Hodge dual identity:

$$D \wedge \widetilde{T}^a := R^a{}_b \wedge q^b. \tag{19.60}$$

This is the identity:

$$D_\mu T^{\kappa\mu\nu} := R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{19.61}$$

which is the following cyclic sum identity:

$$\begin{aligned} &\widetilde{R}^{\lambda}{}_{\rho\mu\nu} + \widetilde{R}^{\lambda}{}_{\mu\nu\rho} + \widetilde{R}^{\lambda}{}_{\nu\rho\mu} \\ &\quad := (\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho})_{HD} \\ &\qquad + (\partial_\nu \Gamma^\lambda_{\rho\mu} - \partial_\rho \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\rho\mu} - \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\nu\mu})_{HD} \\ &\qquad + (\partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\rho\nu})_{HD} \neq 0 \end{aligned} \tag{19.62}$$

where the individual definitions are as follows:

$$\begin{aligned} \widetilde{R}^{\lambda}{}_{\rho\mu\nu} &= (\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\nu\mu})_{HD}, \\ \widetilde{R}^{\lambda}{}_{\mu\nu\rho} &= (\partial_\nu \Gamma^\lambda_{\rho}\mu - \partial_\rho \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\rho\mu} - \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\nu\mu})_{HD}, \\ \widetilde{R}^{\lambda}{}_{\nu\rho\mu} &= (\partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\rho\nu})_{HD}. \end{aligned} \tag{19.63}$$

Acknowledgments

The British Government is thanked for a Civil List Pension for pre-eminent contributions to Britain in science, and the staffs of AIAS and SG for many interesting discussions.

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