Spiral Galaxies and Cartan Torsion

by

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Abstract

The structure and velocity curve of a spiral galaxy is described in terms of a constant Cartan torsion (spinning space-time) using Einstein Cartan Evans (ECE) unified field theory. In the central bulge region gravitational attraction predominates, i.e. the Riemann curvature predominates. In the spiral arms the Cartan torsion predominates. It is shown that the structure of the spiral arms is a hyperbolic spiral due to the underlying constant spinning of space-time. In this generally covariant unified field theory dark matter does not exist and is replaced by Cartan torsion, missing from the Einstein Hilbert field theory of gravitation and its weak field limit, Newtonian dynamics.

Keywords: Einstein Cartan Evans (ECE) field theory, spiral galaxy, velocity curve.

6.1 Introduction

It is well known that standard cosmology is based on the Einstein Hilbert (EH) field theory of gravitation, published in 1916. The weak field limit of EH is the Newtonian theory of gravitation, based on the inverse square law. Recently [1–12] Einstein Cartan Evans (ECE) field theory has
extended EH and developed it into a generally covariant unified field theory which has been applied extensively [1–12] across physics, chemistry and engineering and which has become mainstream physics, replacing the obsolete standard model [15]. In Section 6.2 ECE theory is applied to describe straightforwardly the structure and velocity curve [13] of the well known spiral galaxy [14] in order to illustrate the fact that ECE is the completed theory of general relativity, and as such is simpler and more powerful than obsolete conjectures such as dark matter. Maps of “dark matter” in the universe should be considered as maps of the Cartan torsion (spinning space-time), and a spiral galaxy is a clear and easily observable example of spinning space-time. In EH theory there is only curving space-time, and EH is the limit of Cartan geometry where torsion is zero. EH is therefore severely limited in its application. In Section 6.2 it is shown that the well known, and non-Newtonian, velocity curve of a spiral galaxy is the direct result of a constantly spinning space-time. The constant spin results in a constant orbital velocity in the spiral arms of the galaxy. The angular velocity corresponding to a constant orbital velocity results in the hyperbolic spiral as is straightforward to show. This is matched to experimental data from M101, giving an excellent description of the well known spiral arms of stars which characterize the galaxy. The complete spiral galaxy is therefore made up of a central portion (“central bulge”) in which central gravitational forces predominate and the spiral arms, in which the spinning of space-time predominates. In Section 6.3 some discussion of this result is given in the context of galaxy clusters and super-clusters.

6.2 The Role of Spinning Space-Time in Spiral Galaxies

The galactic rotation curve of a spiral galaxy is shown in Fig. (6.1).

It is a plot of the orbital velocity of stars and other objects against the distance from the center of the galaxy. The result of the Newtonian limit of EH

![Fig. 6.1. Galactic rotation curve for Newtonian region (A) and non-Newtonian region (B).](image-url)
is given as the dotted line, while the ECE result, to be derived in this section, matches the experimental result in region $B$ of Figure 6.1. In region $A$ the EH limit of ECE applies, the limit where the torsion becomes small compared with the curvature (curvature form much bigger than the torsion form). In region $B$ the torsion dominates over the gravitational attraction (torsion form much bigger than the curvature form). Therefore the spiral arms of stars (region $B$) do not collapse, as observed experimentally [14] because the spinning of the underlying space-time dominates over the gravitational pull that would cause the spiral arms to collapse [13, 14]. This is a straightforward explanation of both the structure and dynamics of spiral galaxies without using the obsolete concept of “dark matter”. The main conclusion is that there is no “dark matter” in the universe, the observed dynamics are satisfactorily explained with general relativity provided that the Cartan torsion is fully instated in the theory. The complete geometry of general relativity must include torsion as well as curvature. Einstein and Hilbert did not infer torsion in 1916 because both used Riemann geometry without torsion. Torsion was inferred by Cartan in 1922 who extended Riemann to Cartan or differential geometry. It was not until the advent of ECE theory in 2003 that the importance of torsion to physics was fully realized and developed.

The first step in the derivation of the structure and dynamics of a spiral galaxy is to derive the angular momentum from ECE theory, and then to derive the torque due to the Cartan torsion of space-time. It has been inferred [1–12] that angular momentum in ECE theory is:

$$J^a _\mu = J^{(0)} q^a _\mu$$  \hspace{1cm} (6.1)

where $J^{(0)}$ is a proportionality factor and where $q^a _\mu$ is the unitless Cartan tetrad [15]. The fundamental definition of the tetrad [1–12, 15] then implies that:

$$W^a = q^a _\mu V^\mu$$  \hspace{1cm} (6.2)

where $W^a$ and $V^\mu$ are vectors to be defined. As usual in differential geometry [15] the label $a$ describes the tangent space-time and $\mu$ describes the base manifold. Restricting consideration to the three space indices Eq. (6.2) is:

$$\begin{bmatrix} W^1 \\ W^2 \\ W^3 \end{bmatrix} = \begin{bmatrix} J^1 _1 & J^1 _2 & J^1 _3 \\ J^2 _1 & J^2 _2 & J^2 _3 \\ J^3 _1 & J^3 _2 & J^3 _3 \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}$$  \hspace{1cm} (6.3)
Therefore:

\[
W^1 = J_1^1 V^1 + J_2^1 V^2 + J_3^1 V^3 \\
W^2 = J_1^2 V^1 + J_2^2 V^2 + J_3^2 V^3 \\
W^3 = J_1^3 V^1 + J_2^3 V^2 + J_3^3 V^3
\]

It is well known that angular momentum must be an anti-symmetric tensor, so:

\[
W^1 = J_2^1 V^2 + J_3^1 V^3 \\
W^2 = J_3^2 V^1 + J_1^2 V^3 \\
W^3 = J_1^3 V^1 + J_2^3 V^2
\]

In three dimensions we can write this as:

\[
W^1 = J_{12} V^2 + J_{13} V^3 \\
W^2 = J_{21} V^1 + J_{23} V^3 \\
W^3 = J_{31} V^1 + J_{32} V^2
\]

where as usual:

\[
J_{ij} = \frac{1}{2} \epsilon_{ijk} J_k.
\]

Therefore:

\[
W^1 = J_3 V^2 - J_2 V^3 \\
W^2 = J_3 V^1 - J_1 V^3 \\
W^3 = J_1 V^2 - J_2 V^1
\]

or in vector notation:

\[
W = J \times V.
\]

Now identify:

\[
W = p, V = \frac{r}{r^2}
\]

where \( p \) is orbital momentum and where \( r \) is the distance from the center to a point on the radius [16]. Thus:

\[
p = J \times \frac{r}{r^2}
\]
6.2 The Role of Spinning Space-Time in Spiral Galaxies

which is equivalent to:

\[ \mathbf{J} = \mathbf{r} \times \mathbf{p} \]  \hspace{1cm} (6.12)

using the vector triple product rule [17] of vector analysis. Therefore the familiar definition (6.12) of angular momentum has been derived from Cartan geometry, illustrating the fact that physics is geometry.

The torque (in joules) is derived in ECE theory [1–12] from the torsion (in inverse meters) using:

\[ \mathbf{N}^a(\text{torque}) = cJ^{(0)}T^a(\text{torsion}) \]  \hspace{1cm} (6.13)

where \( c \) is the speed of light, a universal constant of general relativity. The torsion is related to angular momentum using the first Cartan structure equation. In differential form notation:

\[ \mathbf{N}^a = c(d \wedge J^a + \omega^a \wedge J^b). \]  \hspace{1cm} (6.14)

If the torsion is constant (independent of time and distance), then:

\[ d \wedge \mathbf{N}^a = 0. \]  \hspace{1cm} (6.15)

In region \( B \) of Figure 6.1 the torsion dominates, so the motion is pure rotation. It has been shown [1–12] that for pure rotation the spin connection of Eq. (6.14) is dual to the tetrad:

\[ \omega^a \mathbf{b} = -\frac{1}{2}\kappa \epsilon^a \mathbf{b} \mathbf{q} \]  \hspace{1cm} (6.16)

where \( \kappa \) as a wave-number. In this context:

\[ \kappa = \frac{\omega}{v} \]  \hspace{1cm} (6.17)

where \( \omega \) is the angular velocity of the spinning space-time and where \( v \) is the orbital velocity appearing in region \( B \) of Figure 6.1.

Adopting the complex circular basis [1–12]:

\[ \mathbf{N}^{(1)*} = c(\nabla \times \mathbf{J}^{(1)*} - i\kappa \mathbf{q}^{(2)} \times \mathbf{J}^{(3)}) \]

et cyclicum.  \hspace{1cm} (6.18)
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The constant torque is the (6.3) component defined by:

\[ \mathbf{N}^{(3)}_* = \mathbf{N}^{(3)} = -i\kappa q^{(1)} \times \mathbf{J}^{(2)} = c\kappa J^{(0)} k = cmv r k. \] (6.19)

Thus:

\[ \mathbf{N}^{(3)} = cmv k \] (6.20)

because:

\[ \kappa r := 1. \] (6.21)

It is seen that the torque is constant because the orbital velocity \( v \) is constant by observation as in region \( B \) of Fig. (6.1). The Newtonian component:

\[ \nabla \times \mathbf{J}^{(3)} = 0 \] (6.22)

is zero because \( \mathbf{J}^{(3)} \) is in the same (6.3) (or \( Z \)) axis as \( \mathbf{T}^{(3)} \). Eq. (6.20) is exactly analogous to the well known \( \mathbf{B}^{(3)} \) spin field of electrodynamics [1–12] observed in the magnetization of matter by circularly polarized electromagnetic radiation - the inverse Faraday effect. The overall conclusion is that the constant orbital velocity \( v \) is produced by an underlying space-time torque of type (6.20). Such a torque is missing entirely from EH theory, but is clearly observable as a spiral galaxy. This explanation of the completed theory of general relativity is much simpler and more powerful than the elaborate conjectures of “dark matter” theory. ECE is therefore preferred by Ockham’s Razor and comparison with experimental data.

The angular velocity of the spiral galaxy in region \( B \) of Figure 6.1 is [16]:

\[ \omega = \frac{d\theta}{dt} = \frac{v}{r}. \] (6.23)

For constant \( v_0 \) as observed experimentally [13, 14]:

\[ \frac{d\theta}{dt} = \frac{v_0}{r} \] (6.24)

so:

\[ \theta = v_0 \int_0^\tau \frac{dt}{r} = \frac{v_0 \tau}{r} \] (6.25)
which is the equation [18] of the hyperbolic spiral:

\[ r = a^2 \theta \]  (6.26)

where

\[ a = v_0 \tau, n = -1. \]  (6.27)

In Fig. (6.2) the hyperbolic spiral is superimposed on a photograph of the M101 spiral galaxy, and the fit is excellent.

The well known Newtonian result is, in standard notation [16]

\[ \frac{\alpha}{r} = 1 + \epsilon \cos \theta \]  (6.28)

where

\[ v^2 = \frac{k}{\mu} \left( \frac{2}{r} - \frac{1}{a} \right). \]  (6.29)

This is plotted as the dotted line in Fig. (6.1). It is seen that the spiral galaxy is non-Newtonian because the spiral galaxy is due to the spinning of space-time ITSELF. This concept has been neglected in EH theory and has unfortunately led to the introduction of the spurious ideas of “dark matter”.

6.3 Discussion

The complete dynamics of a spiral galaxy are described in ECE theory by the first and second Cartan structure equations [1–12, 15]:

\[ T^a = D \wedge q^a := d \wedge q^a + \omega^a_b \wedge q^b \]  (6.30)
\[ R^a_b = D \wedge \omega^a_b := d \wedge \omega^a_b + \omega^a_c \wedge \omega^c_b \]  (6.31)

where \( T \) and \( R \) are the torsion and Riemann forms respectively. These are inter-related by the first Bianchi identity of differential geometry [15]:

\[ D \wedge T^a := R^a_b \wedge q^b. \]  (6.32)

The curvature form is always governed by the second Bianchi identity:

\[ D \wedge R^a_b := 0. \]  (6.33)
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In region \( B \) of Fig. (6.1), Eq. (6.15) of Section 6.2 means that:

\[
d \wedge T^a = R^a_{\ b} \wedge q^b - \omega^a_{\ b} \wedge T^b = 0 \quad (6.34)
\]
a solution of which [1–12] is:

\[
R^a_{\ b} = -\frac{1}{2} \kappa \epsilon^a_{\ bc} T^c. \quad (6.35)
\]

The Riemann form in region \( B \) is dual to the torsion form, and has therefore become pure rotational in nature. In other words the Riemann form in region \( B \) has become another way of expressing the torsion. In region \( A \) on the other hand the Riemann form is not dual to the torsion because in that region the gravitational force dominates in the central bulge [13, 14] of the galaxy. The gravitational force is centrally directed [16] or translational in nature. This was the only kind of Riemann form considered by Einstein and Hilbert. More generally, (ECE theory), the Riemann form is roto-translational in nature. In the transition from region \( A \) to \( B \) in Fig. (6.1) there is interaction between rotation and translation.

The EH theory is described geometrically [1–12] by:

\[
\omega^a_{\ b} \wedge T^b = 0 \quad (6.36)
\]
because the torsion is zero:

\[
T^b = 0. \quad (6.37)
\]

The Ricci cyclic equation [1–12] of EH theory then demands that:

\[
R^a_{\ b} \wedge q^b = 0 \quad (6.38)
\]
but clearly:

\[
R^a_{\ b} \neq 0. \quad (6.39)
\]

Therefore in EH theory the torque due to spinning space-time itself is missing entirely:

\[
N^a = 0 \quad (6.40)
\]

so that large parts of cosmology are also missing entirely. This paper has illustrated this for a very simple model of a spiral galaxy, but the same principles could be applied in all situations, for example galaxy clusters and super-clusters [13, 14]. ECE is therefore a major advance in cosmology.
6.4 Graphical Representations

Figure 6.2 shows the Galaxy model according to ECE theory. Gravitation dominates the central bulge while torsion dominates the spiral arms. Orbital velocity is tangential to virtual centric circles (dashed line). This picture is observed by astronomy.

In contrast, the resulting picture of the galaxy model according to Newtonian and Einstein-Hilbert theory (Fig. 6.3) is quite different. Gravitation is the only interaction and enforces any spiral arms to move into the central bulge. This result was the reason for inventing “black matter” which is placed outside the galaxy and attracts the arms so they do not fall back into the center.

![Fig. 6.2. Galaxy model according to ECE theory.](image-url)
Fig. 6.3. Galaxy model according to Newtonian and Einstein-Hilbert theory.

Fig. 6.4. Hyperbolic spiral $r = \frac{1}{t}$.

In Fig. (6.4) the hyperbolic spiral

$$r = \frac{1}{t}$$

(6.41)

is represented, where $t = \theta$ is the polar angle. For $t \to 0$ the curve goes to the asymptote $y = 1$. For $t \to \infty$ it spirals against the origin.

Fig. (6.5) shows the logarithmic spiral

$$r = \exp \left( \frac{t}{2} \right).$$

(6.42)
6.4 Graphical Representations

Fig. 6.5. Logarithmic spiral $r = exp(0.5t)$.

Fig. 6.6. Picture of galaxy M101 from Hubble telescope.
Fig. 6.7. Hyperbolic spirals fitted to the arms of galaxy M101.

For \( t \to -\infty \) the curve spirals against the origin. For \( t \to +\infty \) there is no asymptote.

Fig. (6.6) is a picture of galaxy M101 from the Hubble telescope. Several hyperbolic spirals have been fitted to the arms of this galaxy. This is shown in Fig. (6.7). A graphical linear stretching of the spirals was applied in the range 100-200 %.

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