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The Effect of Gravitation on the Inverse Faraday Effect and Faraday Effect: Multiple Field Interactions in ECE Theory

by

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Abstract

The effect of gravitation on the inverse Faraday effect and Faraday effect is considered as an example of multiple field interactions in Einstein Cartan Evans (ECE) field theory. The interactions of fundamental fields are considered on classical, semi-classical and fully quantized levels. For example, the fully quantized interaction between electron, photon and graviton is considered. In its classical limit, the interaction of photon and graviton is shown to produce the light deflection due to gravitation.

Keywords: Einstein Cartan Evans (ECE) field theory, inverse Faraday effect, Faraday effect, gravitation, multiple field interaction in ECE theory.

13.1 Introduction

Recently, a generally covariant unified field theory has been developed [1–12] from standard Cartan geometry [13, 14]. It is known as Einstein Cartan Evans (ECE) field theory and has several fundamental advantages (www.aias.us) over the standard model. One major advantage is the ability of ECE theory to describe multiple field interactions on classical, semi-classical and quantum levels. In this paper an example of a multiple field interaction is given - that between the fermion matter field (exemplified by an electron), the electromagnetic field, and the gravitational field. In Section 13.2 this three field interaction is developed on the classical level and exemplified by the effect of gravitation on the inverse Faraday effect (IFE) and Faraday effect (FE). In Section 13.3, the difference is emphasized between the motion of an electron in a static magnetic field and an electromagnetic field. In Section 13.4 the self consistency of the classical method is checked by comparing a direct integration method with a Hamilton Jacobi method. In Section 13.5 the calculation is extended to the quantum level by considering the interaction between an electron and photon using ECE wave equations. In Section 13.6 the electron photon graviton interaction is considered on the quantum level, and in Section 13.7 the light deflection due to gravitation is obtained straightforwardly on the classical level.

13.2 Classical Limit: The Effect of Gravitation on IFE and FE

In the first approximation the effect of gravitation on the IFE and FE can be developed from previous work on the ECE theory of IFE and FE by using the rule [1–12]:

$$m \rightarrow \frac{\hbar}{e}(kT)^{\frac{1}{2}} \tag{13.1}$$

where m is the electron mass, $\hbar$ is the reduced Planck constant, c is the speed of light, k is the Einstein constant and T is the scalar canonical energy momentum density. This rule originates in the correspondence principle applied to the ECE wave equation [1–12]:

$$(\Box + kT)q^a_\mu \tag{13.2}$$

where q^a_μ is the tetrad wave-function and also the fundamental field. In general this is a unified field, so T and q^a_μ contain information about the interaction of any or all of the fundamental fields of physics: gravitational, electromagnetic, weak, strong and matter fields. If the fermion field is considered for example, it becomes free of the influence of any other fundamental field when:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \tag{13.3}$$

from which follows Eq. (13.1). When the fermion (e.g. electron) is free of any other field its mass is m. In the presence of gravitation for example its mass changes according to Eq. (13.1) and in general its mass changes in the presence of any other field, including the electromagnetic field. The fermion (electron) is no longer free because it is influenced by a gravitational field. To consider the effect of gravitation on the IFE and FE requires a three field interaction: the effect of gravitation on a fermion interacting with the electromagnetic field. The gravitational influence may be developed in a series of approximations. In the first approximation it may be assumed that the electromagnetic field is not affected by the gravitational field, and in this approximation Eq. (13.1) is used with the minimal prescription [1–12, 15]. In a better approximation, developed in later sections of this paper, the fermion and electromagnetic fields are both affected by the gravitational field. On the fully quantum level this requires the simultaneous solution of three ECE wave equations.

In the classical minimal prescription of ECE field theory, the complex valued four potential may be defined by:

$$A_\mu := A_\mu^{(0)} + A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} \tag{13.4}$$

where:

$$a = (0), (1), (2), (3) \tag{13.4a}$$

are polarization indices. The time-like index is (0), the three space-like indices by (1), (2) and (3). Here (1) and its complex conjugate (2) are transverse and (3) is longitudinal. The potential in ECE theory is manifestly covariant, so that all four indices are physical. It is also possible to consider individual components of A_μ in the minimal prescription. An example is the transverse plane wave:

$$\boldsymbol{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi_0} \tag{13.5}$$

where

$$\phi_0 = \omega t - \kappa Z \tag{13.6}$$

is the electromagnetic phase. Here ω is the angular frequency at instant t and κ the wave-number at point Z. In general the four-potential is:

$$A^\mu = \left(\frac{\phi}{c}, \boldsymbol{A}\right) \tag{13.7}$$

where ϕ is the scalar potential and $\boldsymbol{A}$ the vector potential. In the first approximation defined already, ϕ and $\boldsymbol{A}$ are not changed, but m is changed wherever it occurs using Eq. (13.1). In previous work [1–12] it has been shown from direct integration of the classical Einstein equation with minimal prescription that the relevant kinematics of the IFE and FE for the free classical electron (the classical limit of the Dirac electron) are as follows. The angular momentum is:

$$\begin{aligned}\boldsymbol{J} = &\gamma \boldsymbol{r}_0 \times \boldsymbol{p} + e\boldsymbol{r}_0 \times \boldsymbol{A} \\ &+ e\int \boldsymbol{A} dt \times \boldsymbol{v}_0 + \frac{\boldsymbol{e}^2}{\gamma m}\int \boldsymbol{A} dt \times \boldsymbol{A}\end{aligned} \tag{13.8}$$

where $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ are respectively the initial position and velocity of the electron, e is the magnitude of the charge on the electron, m is the mass of the electron, $\boldsymbol{A}$ is the electromagnetic vector potential, and:

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} \tag{13.9}$$

where u is the constant speed of one frame moving with respect to another in a Lorentz boost [13, 15, 16]. For a plane wave such as Eq. (13.5) an analytical solution may be obtained for the magnitude of the angular momentum:

$$J = \left(r_0 + \frac{eA}{\gamma m\omega}\right)(\gamma m v_0 + eA)\,. \tag{13.10}$$

The kinetic energy of the electron is [1–12]:

$$T = \frac{(\gamma m v_0 + eA)^2 c^2}{mc^2(1+\gamma) + e\phi} \tag{13.11}$$

and its angular velocity is:

$$\Omega = \frac{(\gamma m v_0 + eA)c^2}{\left(r_0 + \frac{eA}{\gamma m\omega}\right)(mc^2(1+\gamma) + e\phi)} \tag{13.12}$$

from which the angular velocity of the electromagnetic field can be expressed in terms of Ω as:

$$\omega = \frac{eA\Omega}{\gamma m(x - \Omega r_0)} \tag{13.13}$$

where the factor x is:

$$x = \frac{(\gamma m v_0 + eA)c^2}{mc^2(1+\gamma) + e\phi}. \tag{13.14}$$

Therefore all these kinematic equations are affected by gravitation according to Eq. (13.1), i.e. the electron mass is changed by gravitation wherever it occurs, and in consequence the kinematic quantities are all affected by gravitation. This shows that both the IFE and FE are effected by gravitation.

In a better approximation the effect of gravitation on the minimal prescription itself is considered. Therefore A^μ as well as p^μ is changed by the presence of the gravitational field. Therefore gravitation changes $\boldsymbol{A}$ and ϕ as follows:

$$\boldsymbol{A} \rightarrow \left(\frac{\hbar}{mc} (kT)^{\frac{1}{2}} \right) \boldsymbol{A}, \tag{13.15}$$

$$\boldsymbol{\phi} \rightarrow \left(\frac{\hbar}{mc} (kT)^{\frac{1}{2}} \right) \boldsymbol{\phi}. \tag{13.16}$$

In ECE theory the relation between the electromagnetic potential and the electromagnetic field is also changed by gravitation because the latter implies the existence of a non-zero homogeneous current. In the standard notation of differential geometry [13], the homogeneous current is defined by:

$$j^a := \frac{A^{(0)}}{\mu_0} (R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b) \tag{13.17}$$

where μ_0 is the vacuum magnetic permeability (S.I. units), R^a_b is the curvature form, ω is the spin connection and T^b is the torsion form [1–12]. Therefore various levels of approximation may be used to describe field interactions in ECE theory. The whole of physics can be defined as the interaction of fields. ECE allows this to be considered in a generally covariant manner as demanded by the basics of relativity. The latter is by far the most precise theory in physics, so the ECE methods are well founded in experiment.

13.3 The Motion of the Classical Electron in a Static Magnetic Field and a Radiated Electromagnetic Field

Before proceeding to higher levels of approximation in multiple field interactions the differences between the motion of an electron in a static magnetic field and in a radiated electromagnetic field are emphasized here on the classical level. This is to emphasize that a classical magnetic field has important differences from the radiated electromagnetic spin field of ECE theory. The latter is an observable of IFE and FE and is one of the indications that electrodynamics is a generally covariant sector of unified field theory. The standard model description of a static magnetic field must be developed in ECE theory [1–12] to include the spin connection, but the standard model description is given here for the sake of illustrating the differences in electron motion.

From the minimal prescription the angular momentum of the electron in the static magnetic field is:

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} = e\boldsymbol{r} \times \boldsymbol{A} \tag{13.18}$$

where, in the standard model, the static magnetic flux density is:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A},\ \boldsymbol{A} = \frac{1}{2}\boldsymbol{B} \times \boldsymbol{r}. \tag{13.19}$$

Therefore the electron's angular momentum magnitude is:

$$J = erA = er^2B \tag{13.20}$$

and its kinetic energy is:

$$T = \frac{e^2A^2}{2m}. \tag{13.21}$$

Its angular velocity is:

$$\Omega = 2\frac{T}{J} = \frac{eA}{rm} = \frac{e}{m}B. \tag{13.22}$$

Therefore its angular momentum magnitude can be written as:

$$J = e\Phi \tag{13.23}$$

where:

$$\Phi = r^2B \tag{13.24}$$

is the magnetic flux in weber. The magnetic flux density is $\boldsymbol{B}$ (in tesla or weber per square meter). The quantum of flux is therefore:

$$\Phi = \frac{\hbar}{e}. \tag{13.25}$$

The magnetic dipole moment induced by a static magnetic field in the standard model is therefore

$$\boldsymbol{m} = \frac{e}{2m}\boldsymbol{J} = \left(\frac{e^2r^2}{2m}\right)\boldsymbol{B} \tag{13.26}$$

where

$$\chi := \frac{e^2 r^2}{2m} \tag{13.27}$$

is the static magnetic susceptibility.

It is seen from a comparison with Section 13.2 that these quantities are different from the corresponding ones for an electron in an electromagnetic field. Using the Hamilton Jacobi method for example [1–12] the angular momentum of an electron in a circularly polarized electromagnetic field may be expressed as:

$$\boldsymbol{J}^{(3)} = \frac{e^2 c^2}{\omega^2} \left(\frac{B^{(0)}}{(m^2\omega^2 + e^2 B^{(0)2})^{\frac{1}{2}}} \right) \boldsymbol{B} \tag{13.28}$$

where $\boldsymbol{B}^{(3)}$ is the ECE spin field [1–12]. The latter originates in the spinning of space-time itself and is a radiated field. The static magnetic field is not a radiated field. The angular velocity of the electron in the electromagnetic field from the Hamilton Jacobi method is [1–12] is:

$$\Omega = \left(\omega^2 + \left(\frac{eB^{(0)}}{m} \right)^2 \right)^{\frac{1}{2}}. \tag{13.29}$$

From Eq. (13.8), the angular momentum of the electron in a radiated plane wave is:

$$\boldsymbol{J}^{(3)} = -\frac{ie^2}{\gamma m} \int \boldsymbol{A}^{(1)} dt \times \boldsymbol{A}^{(2)} \tag{13.30}$$

where:

$$\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*} \tag{13.31}$$

in which * denotes complex conjugate. So:

$$\boldsymbol{J}^{(3)} = \frac{e^2 A^{(0)2}}{\gamma m \omega} \boldsymbol{e}^{(3)}. \tag{13.32}$$

Using the fundamental optical relation [17]:

$$A^{(0)} = \frac{c}{\omega} B^{(0)} \tag{13.33}$$

and:

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)} = B^{(0)}\boldsymbol{k} \tag{13.34}$$

it is found that the angular momentum from the direct integration method [1–12] is:

$$\boldsymbol{J}^{(3)} = \frac{e^2c^2}{\gamma m\omega^3}B^{(0)}\boldsymbol{B}^{(3)} \tag{13.35}$$

In the non-relativistic limit:

$$\gamma \to 1,\ m\omega \gg eB^{(0)} \tag{13.36}$$

and Eq. (13.35) becomes Eq. (13.28) of the Hamilton Jacobi method. A fuller comparison of the Hamilton Jacobi method and the direct integration method of describing the relativistic motion of an electron in an electromagnetic field is given in the next section. From Eq. (13.35) the angular momentum magnitude is

$$J = \frac{e^2c^2}{\gamma m\omega^3}B^{(0)2} = \frac{\mu_0 e^2 c}{\gamma m\omega}\left(\frac{I}{\omega^2}\right) \tag{13.37}$$

where [17]:

$$I = \frac{c}{\mu_0}B^{(0)2} \tag{13.38}$$

is the power density in watts per square meter of the electromagnetic field. The induced magnetic dipole moment due to the $\boldsymbol{B}^{(3)}$ spin field is:

$$\boldsymbol{m}^{(3)} = \left(\frac{e^3c^2}{2\gamma m^2\omega^3}\right)B^{(0)}\boldsymbol{B}^{(3)} \tag{13.39}$$

where

$$\beta := \frac{e^3c^2}{2\gamma m^2\omega^3} \tag{13.40}$$

is the magnetic hyper-magnetizability of one electron. It is seen that the $\boldsymbol{B}^{(3)}$ field interacts through this property while the static magnetic field interacts through the one electron static susceptibility.

13.4 Comparison of Direct Integration and Hamilton Jacobi Methods

The angular momentum magnitude from the direct integration method gives:

$$J = \frac{1}{\gamma m\omega}(\gamma m\omega r_0 + eA^{(0)})(\gamma m v_0 + eA^{(0)}) \tag{13.41}$$

where r_0 and v_0 are the initial position and velocity of the electron. The Hamilton Jacobi method [1–12] gives:

$$J_{\text{int}} = \frac{ce^2 A^{(0)2}}{\omega(m^2c^2 + e^2A^{(0)2})^{\frac{1}{2}}}. \tag{13.42}$$

The interaction terms of Eq. (13.41) are:

$$J_{\text{int}} = eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2A^{(0)2}}{\gamma m\omega} \tag{13.43}$$

which can be compared directly with Eq. (13.42).

In the non-relativistic limit:

$$\gamma \to 1,\ mc \gg eA^{(0)}, \tag{13.44}$$

and both Eqs. (13.42) and (13.43) give:

$$J_{\text{int}} \to \frac{e^2A^{(0)2}}{m\omega}. \tag{13.45}$$

So in this limit both methods give the same result. In the opposite hyper-relativistic limit:

$$eA^{(0)} \gg mc \tag{13.46}$$

Eq. (13.42) becomes:

$$J_{\text{int}} \to \frac{ecA^{(0)}}{\omega}. \tag{13.47}$$

If the second order term in Eq. (13.43) is neglected then:

$$J_{\text{int}} \sim eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) \tag{13.48}$$

so:

$$r_0 + \frac{v_0}{\omega} \to \frac{c}{\omega} \tag{13.49}$$

i.e.

$$v_0 \to c, \ \frac{v_0}{\omega} \gg r_0 \tag{13.50}$$

in the hyper-relativistic limit.

In general, comparing Eqs. (13.42) and (13.43):

$$eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2A^{(0)2}}{\gamma m\omega} = \frac{ce^2A^{(0)2}}{\omega(m^2c^2 + e^2A^{(0)2})^{\frac{1}{2}}}. \tag{13.51}$$

For an initially stationary electron at the origin:

$$v_0 = 0, \ r_0 = 0 \tag{13.52}$$

we obtain:

$$\gamma = \frac{1}{mc}(m^2c^2 + e^2A^{(0)2})^{\frac{1}{2}} \tag{13.53}$$

which is precisely the expression used in the original Hamilton Jacobi method of volume one of "The Enigmatic Photon" [1–12]. In these equations the factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{13.54}$$

is used to find that:

$$1 - \frac{v^2}{c^2} = \left(1 + \left(\frac{eA^{(0)}}{mc}\right)^2\right)^{-1}. \tag{13.55}$$

For $v \ll c$:

$$v \sim \frac{e^2A^{(0)2}}{mc^2} \tag{13.56}$$

i.e. if

$$eA^{(0)} \ll mc. \tag{13.57}$$

Eq. (13.56) means that the initially stationary electron has attained an orbital velocity of $\frac{e^2A^{(0)2}}{mc^2}$ from the applied electromagnetic field. This is therefore a self-consistent analysis. The important result for the IFE and radiatively induced fermion resonance (RFR) [1–12] is Eq. (13.45). Therefore IFE and RFR have been self-consistently derived in many ways in this paper and in previous work [1–12], from classical to fully quantum levels. These theoretical methods assume a circularly polarized plane wave of type (5), so must be tested experimentally under the same conditions, using a circularly polarized radio frequency plane wave and an electron beam. In the non-relativistic limit:

$$T \to \frac{e^2A^{(0)2}}{2m} \tag{13.58}$$

and comparing Eqs. (13.45) and (13.58):

$$T = \frac{1}{2}\omega J = \frac{1}{2}\Omega J \tag{13.59}$$

so in the non-relativistic limit:

$$\Omega = \omega \tag{13.60}$$

i.e. the angular velocity ω of the electromagnetic field is fully imparted to the electron. This same result is obtainable from Eq. (13.29) of the Hamilton Jacobi method [1–12]. RFR is then derived from Eq. (13.58) using the SU(2) basis [1–12] to give:

$$\hbar\omega_{\text{res}} = \frac{e^2A^{(0)2}}{m} \tag{13.61}$$

as the difference of energy levels of the Z Pauli matrix as in ESR or NMR. It is therefore deduced that gravitation will also affect the RFR resonance frequency because RFR is the detection of IFE using resonance instead of induction, and as argued already, IFE is affected by gravitation.

In the SU(2) basis the interaction kinetic energy is:

$$\begin{aligned} T &= \frac{1}{2m}\boldsymbol{\sigma}\cdot(\boldsymbol{p}+e\boldsymbol{A})\boldsymbol{\sigma}\cdot(\boldsymbol{p}+e\boldsymbol{A}^*) \\ &= \frac{p^2}{2m} + \frac{ie^2}{2m}\boldsymbol{\sigma}\cdot\boldsymbol{A}\times\boldsymbol{A}^* + \cdots \end{aligned} \tag{13.62}$$

where $\boldsymbol{A}^*$ is the complex conjugate of $\boldsymbol{A}$. The RFR term is then given by:

$$\hbar\omega_{\text{res}} = \frac{2e^2}{2m}i|\boldsymbol{A}\times\boldsymbol{A}^*|. \tag{13.63}$$

The resonance occurs between the states of:

$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{13.64}$$

and introduces the factor 2 in the numerator of Eq. (13.63), as in ESR theory. The conjugate product is:

$$i\boldsymbol{A} \times \boldsymbol{A}^* = A^{(0)2}\boldsymbol{k} \tag{13.65}$$

so:

$$i\boldsymbol{\sigma} \cdot \boldsymbol{A} \times \boldsymbol{A}^* = A^{(0)2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{13.66}$$

A photon $\hbar\omega_{\text{res}}$ of a probe beam is absorbed from the interaction kinetic energy:

$$T = \begin{bmatrix} \frac{e^2 A^{(0)2}}{2m} & 0 \\ 0 & -\frac{e^2 A^{(0)2}}{2m} \end{bmatrix} = \begin{bmatrix} T_{\text{spin up}} & 0 \\ 0 & T_{\text{spin down}} \end{bmatrix}. \tag{13.67}$$

Thus:

$$\begin{aligned} &T_{\text{spin up}} - T_{\text{spin down}} = \hbar\omega_{\text{res}} \\ &= \frac{e^2 A^{(0)2}}{2m} - \left(-\frac{e^2 A^{(0)2}}{2m} \right) = \frac{e^2 A^{(0)2}}{m}. \end{aligned} \tag{13.68}$$

So the RFR frequency is:

$$f_{\text{res}} = \frac{\omega_{\text{res}}}{2\pi} = \frac{e^2 A^{(0)2}}{2\pi\hbar m}. \tag{13.69}$$

At this frequency the probe beam's photon $\hbar\omega_{res}$ is absorbed, and one absorption line is observed in a spectrometer. Using Eqs. (13.33) and (13.38) the RFR resonance frequency can be expressed in terms of the pump beam's power density I in watts per square meter:

$$f_{res} = \left(\frac{e^2 \mu_0 c}{2\pi\hbar m} \right) \frac{I}{\omega^2}. \tag{13.70}$$

The pump beam should be a circularly polarized, radio frequency, plane wave for accurate comparison with this theory, and the sample should be N electrons in an electron beam. The great advantage of RFR over ESR and NMR is that RFR does not use magnets and by adjusting I and ω, has a much greater spectral resolution. RFR also has its own chemical shift pattern [1–12], so if developed would be a powerful new fermion resonance technique.

The Clifford algebra needed to prove:

$$\boldsymbol{\sigma}\cdot\boldsymbol{A}\;\boldsymbol{\sigma}\cdot\boldsymbol{A}^{*}=\boldsymbol{A}\cdot\boldsymbol{A}^{*}\sigma_0+i\boldsymbol{A}\times\boldsymbol{A}^{*}\cdot\sigma_Z\boldsymbol{k} \tag{13.71}$$

is as follows. For plane waves:

$$A_Z=0 \tag{13.72}$$

so:

$$\begin{aligned}\boldsymbol{\sigma}\cdot\boldsymbol{A}\;\boldsymbol{\sigma}\cdot\boldsymbol{A}^{*}&=\begin{bmatrix}0 & A_X-iA_Y\\ A_X+iA_Y & 0\end{bmatrix}\begin{bmatrix}0 & A_X^{*}-iA_Y^{*}\\ A_X^{*}+iA_Y^{*} & 0\end{bmatrix}\\&=A^{(0)2}\begin{bmatrix}1&0\\0&1\end{bmatrix}+A^{(0)2}\begin{bmatrix}1&0\\0&-1\end{bmatrix}\end{aligned} \tag{13.73}$$

where:

$$\sigma_0=\begin{bmatrix}1&0\\0&1\end{bmatrix},\;\sigma_Z=\begin{bmatrix}1&0\\0&-1\end{bmatrix} \tag{13.74}$$

Q.E.D.

13.5 Interaction of Photon and Electron

The ECE equations for the interaction [1–12] are:

$$(\Box+kT)q_\mu^a=0 \tag{13.75}$$

$$(\Box+kT)A_\mu^a=0 \tag{13.76}$$

where q_μ^a is the tetrad wave-function of the electron and where A_μ^a is that of the photon. These are generally covariant equations of quantum mechanics. Denote the electron momentum by p^μ and the photon momentum by π^μ. In Eq. (13.75):

$$\Box=-\frac{1}{\hbar^2}p^\mu p_\mu \tag{13.77}$$

and in Eq. (13.76):

$$\square = -\frac{1}{\hbar^2}\pi^\mu \pi_\mu \tag{13.78}$$

as a result of the operator equivalence rules:

$$p^\mu = i\hbar\partial^\mu, \ \pi^\mu = i\hbar\partial^\mu \tag{13.79}$$

which derive from wave particle dualism. For a free electron:

$$\left(\square + \left(\frac{m_e c}{\hbar}\right)^2\right) q^a_\mu = 0, \tag{13.80}$$

$$\square = -\frac{1}{\hbar^2} p^\mu p_\mu, \tag{13.81}$$

and for a free photon:

$$\left(\square + \left(\frac{m_p c}{\hbar}\right)^2\right) A^a_\mu = 0, \tag{13.82}$$

$$\square = -\frac{1}{\hbar^2}\pi^\mu \pi_\mu, \tag{13.83}$$

where m_e and m_p are respectively the masses of the electron and photon. On the classical level the interaction of the photon and electron is:

$$p^\mu_{\text{free}} + \pi^\mu_{\text{free}} = p^\mu_{\text{free}} + eA^\mu + \pi^\mu_{\text{free}} - eA^\mu \tag{13.84}$$

so that total energy/momentum is conserved. The electron momentum increases by:

$$p^\mu \rightarrow p^\mu + eA^\mu \tag{13.85}$$

and the photon momentum decreases by:

$$\pi^\mu \rightarrow \pi^\mu - eA^\mu. \tag{13.86}$$

In general A^μ is complex valued [1–12], so:

$$\Box_e \rightarrow -\frac{1}{\hbar^2}(p^\mu + eA^\mu)(p_\mu + eA^*_\mu) \tag{13.87}$$

$$\Box_p \rightarrow -\frac{1}{\hbar^2}(\pi^\mu - eA^\mu)(\pi_\mu - eA^*_\mu). \tag{13.88}$$

Substituting in Eqs. (13.75) and (13.76) and averaging to terms to first order for an oscillatory electromagnetic field the following simultaneous wave equations are obtained:

$$\left(\Box_e - \frac{e^2 A^{(0)2}}{\hbar^2} + \left(\frac{m_e c}{\hbar}\right)^2\right) q^a_\mu = 0 \tag{13.89}$$

$$\left(\Box_p - \frac{e^2 A^{(0)2}}{\hbar^2} + \left(\frac{m_p c}{\hbar}\right)^2\right) A^a_\mu = 0. \tag{13.90}$$

So the field interaction is described by:

$$(kT)_e = \frac{1}{\hbar^2}(m_e^2 c^2 - e^2 A^{(0)2}) \tag{13.91}$$

$$(kT)_p = \frac{1}{\hbar^2}(m_p^2 c^2 - e^2 A^{(0)2}). \tag{13.92}$$

For the electromagnetic field:

$$\left(\Box_p + \left(\frac{m_p c}{\hbar}\right)^2\right) A^a_\mu = \left(\frac{e^2 A^{(0)2}}{\hbar^2}\right) A^a_\mu \tag{13.93}$$

and for the fermionic field:

$$\left(\Box_e + \left(\frac{m_e c}{\hbar}\right)^2\right) q^a_\mu = \left(\frac{e^2 A^{(0)2}}{\hbar^2}\right) q^a_\mu. \tag{13.94}$$

These are the generally covariant equations of quantum electrodynamics describing the interaction of a photon and electron in general relativity as represented by ECE theory. Their classical equivalents are:

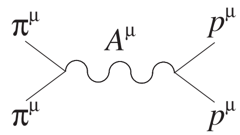
$$p^\mu p_\mu = m_e^2 c^2 - e^2 A^{(0)2} \tag{13.95}$$

$$\pi^\mu \pi_\mu = m_p^2 c^2 - e^2 A^{(0)2} \tag{13.96}$$

i.e.:

$$p^\mu p_\mu - m_e^2 c^2 = \pi^\mu \pi_\mu - m_p^2 c^2 = -e^2 A^{(0)2}. \tag{13.97}$$

The interaction of the photon and electron with gravitation requires further terms in kT. The above is summarized in the diagram:



(13.98)

but unlike a Feynman diagram of special relativity, this is a diagram of general relativity.

13.6 Electron Photon Graviton Interaction

On the fully quantized level the triple interaction is represented by:

$$(p_1 + p_2) + p_5 = (p_3 + p_4) + p_6 \tag{13.99}$$

where:

$$p_3 = p_1 + A + g \tag{13.100}$$
$$p_4 = p_2 - A + g \tag{13.101}$$
$$p_6 = p_5 - 2g \tag{13.102}$$

So:

$$p_1 + p_2 + p_5 = p_1 + A + g + p_2 - A + g + p_5 - 2g. \tag{13.103}$$

In this scheme the electron has gained momentum from both the photon and the graviton. The photon loses A (indexless notation) but gains g from the quantized gravitational field. The graviton loses g to both the electron and photon, so loses a total of $2g$. The free particle equations are Eqs. (13.80) and (13.82) of Section 13.5 and the wave equation of the gravitational field:

$$\left(\Box + \left(\frac{m_g c}{\hbar} \right)^2 \right)^2 G_\mu^a = 0, \tag{13.104}$$

$$\Box = -\frac{1}{\hbar^2} g^\mu g_\mu \tag{13.105}$$

where G^a_μ is a tetrad wave-function. The triple interaction is therefore:

$$p^\mu \to p^\mu + eA^\mu + \epsilon G^\mu \tag{13.106}$$

$$\pi^\mu \to \pi^\mu - eA^\mu + \epsilon G^\mu \tag{13.107}$$

$$g^\mu \to g^\mu - 2\epsilon G^\mu \tag{13.108}$$

on the classical level and:

$$(i\hbar\partial^\mu + eA^\mu + \epsilon G^\mu)(i\hbar\partial_\mu + eA^*_\mu + \epsilon G^*_\mu)q^a_\mu = m_e^2 e^2 q^a_\mu \tag{13.109}$$

$$(i\hbar\partial^\mu - eA^\mu + \epsilon G^\mu)(i\hbar\partial_\mu - eA^*_\mu + \epsilon G^*_\mu)A^a_\mu = m_p^2 c^2 A^a_\mu \tag{13.110}$$

$$(i\hbar\partial^\mu - 2\epsilon G^\mu)(i\hbar\partial_\mu - 2\epsilon G^*_\mu)g^a_\mu = m_g^2 c^2 g^a_\mu \tag{13.111}$$

on the quantum level. Here ϵ is a proportionality that plays the role of e for the gravitational field. By definition:

$$A_\mu := A^{(0)}_\mu + A^{(1)}_\mu + A^{(2)}_\mu + A^{(3)}_\mu \tag{13.112}$$

$$G_\mu := G^{(0)}_\mu + G^{(1)}_\mu + G^{(2)}_\mu + G^{(3)}_\mu . \tag{13.113}$$

Eqs. (13.109) to (13.111) must be solved as simultaneous equations. In the classical limit these simultaneous wave equations become simultaneous equations of motion:

$$(p^\mu + eA^\mu + \epsilon G^\mu)(p_\mu + eA^*_\mu + \epsilon G^*) = m_e^2 c^2 \tag{13.114}$$

$$(\pi^\mu - eA^\mu + \epsilon G^\mu)(\pi_\mu - eA^*_\mu + \epsilon G^*_\mu) = m_p^2 c^2 \tag{13.115}$$

$$(g^\mu - 2\epsilon G^\mu)(g_\mu - 2\epsilon G^*_\mu) = m_g^2 c^2 . \tag{13.116}$$

In the standard model the photon and graviton masses are zero and all that is usually considered is:

$$(p^\mu + eA^\mu)(p_\mu + eA_\mu) = m_e^2 c^2. \tag{13.117}$$

ECE theory provides a wholly new dimension and allows such problems as photon graviton interaction to be considered using the simultaneous equations:

$$(\pi^\mu + \epsilon G^\mu)(\pi_\mu + \epsilon G^*_\mu) = m_p^2 c^2, \tag{13.118}$$

$$(g^\mu - \epsilon G^\mu)(g_\mu - \epsilon G^*_\mu) = m_g^2 c^2. \tag{13.119}$$

In the standard model this method cannot be used because the basic wave equation of gravitation is missing. This is the ECE wave equation:

$$(\Box + kT) g^a_\mu = 0 \tag{13.120}$$

where the gravitational wave-function is the tetrad g^a_μ. In this method the light deflection due to gravitation for example is:

$$\theta = \int \frac{v}{r} dt. \tag{13.121}$$

The most general wave equations for the triple interaction are:

$$(\Box + kT) q^a_\mu = 0 \tag{13.122}$$

$$(\Box + kT) A^a_\mu = 0 \tag{13.123}$$

$$(\Box + kT) g^a_\mu = 0. \tag{13.124}$$

For example, for the electron the expression for kT is derived from Eq. (13.109):

$$\left(\partial^\mu - \frac{ie}{\hbar} A^\mu - \frac{i\epsilon}{\hbar} G^\mu\right)\left(\partial_\mu - \frac{ie}{\hbar} A^*_\mu - \frac{i\epsilon}{\hbar} G^*_\mu\right) q^a_\mu = -\frac{m_e^2 c^2}{\hbar^2} q^a_\mu \tag{13.125}$$

so:

$$(kT)_{\text{electron}} = \frac{m_e^2 c^2}{\hbar^2} - \frac{1}{\hbar^2}(eA^\mu + \epsilon G^\mu)(eA_\mu^* + \epsilon G_\mu^*) \\ - \frac{i}{\hbar}((eA^\mu + \epsilon G^\mu)\partial_\mu + \partial^\mu(eA_\mu^* + \epsilon G_\mu^*)) \tag{13.126}$$

where:

$$p^\mu = i\hbar\partial^\mu. \tag{13.127}$$

In the laboratory:

$$|eA^\mu| \gg |\epsilon G^\mu| \tag{13.128}$$

by many orders of magnitude. So for all practical purposes and after averaging:

$$(kT)_{\text{electron}} = \frac{1}{\hbar^2}\left(me^2c^2 - e^2 A^\mu A_\mu^*\right) \tag{13.129}$$

In a cosmological context, light bending by gravitation is observed, and also many other effects of gravitation on light, such as polarization changes [1–12], Eqs. (13.118) and (13.119) predict that there is an inverse Faraday effect on light caused by gravitation.

13.7 Calculation of Light Deflection Due to Gravitation

The relevant semi-classical ECE wave equation leads to:

$$(\pi^\mu + \epsilon G^\mu)(\pi_\mu + \epsilon G_\mu^*) = m_p^2 c^2 \tag{13.130}$$

where π^μ is the photon energy-momentum and m_p the photon mass. The gravitational interaction is given by ϵG^μ, where ϵ is to be determined. Adopting the results of earlier sections of this paper the interaction kinetic energy is:

$$T_{\text{int}} = \frac{(G^{(0)2})c^2}{mc^2(1+\gamma) + \epsilon c G^{(0)}} \tag{13.131}$$

in the limit:

$$\epsilon G^{(0)} \gg mc(1+\gamma). \tag{13.132}$$

Now identify:

$$T_{\text{int}} = \epsilon c G^{(0)} = \frac{4GmM}{r} \tag{13.133}$$

where G is Newton's constant and M is the mass of a particle that attracts the photon mass. Here r is the distance between the particles. By units analysis:

$$T_{\text{int}} = \omega J \tag{13.134}$$

where ω is angular velocity and J is angular momentum. So:

$$\omega = \frac{d\theta}{dt} = \frac{4GmM}{r}\frac{1}{J} \tag{13.135}$$

$$\theta = \frac{4GmM}{r}\int \frac{1}{J} dt. \tag{13.136}$$

Now identify:

$$\int \frac{1}{J} dt = \frac{1}{mc^2} = \frac{1}{\hbar\omega} \tag{13.137}$$

to find:

$$\theta = \frac{4GM}{rc^2} \tag{13.138}$$

which is the observed relativistic light deflection due to gravity. It has been assumed that:

$$J = \hbar, \int dt = \frac{1}{\omega} \tag{13.139}$$

and the de Broglie equation has also been used:

$$\hbar\omega = mc^2. \tag{13.140}$$

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